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Reliability Analysis of Simply Supported Steel Beams

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Abstract: Reliability analysis of simply supported steel beam designed in accordance with BS5950 (1990) was carried using First Order Reliability Method (FORM), at ultimate and serviceability limit states. Design variables such as load ratio, span and intensity of imposed loading were considered random and stochastic. It was shown among other findings that, when the span (L) of the beam and the load ratio (α_o) are kept constant, as the magnitude of imposed load (Q_k) increased by 200%, the safety of the designed section decreased by 66% considering bending, 84% considering shear and by 82% when deflection criterion was considered. Also, the weighted average safety indices are respectively 2.03, 5.69 and 1.72 for bending, shear and deflection failure criteria of the BS5950 (1990). Therefore, BS5950 design results seems conservative with respect to shear, unsafe with respect to bending (under low shear load), and satisfactory with respect to deflection.

Key words: Reliability, steel beams, safety index, structural design

INTRODUCTION

The aim of the structural designer is to produce design and drawings for a safe and economical structure that fulfills its intended purpose (MacGinley, 1998).

Structural design is accomplished by computing the internal forces and moment acting on each component of the structure, followed by the selection of appropriate cross section for a given grade of steel. The stresses in the structure are generally computed by the theory of elasticity. However, the load carrying capacity of the steel structure depends mostly upon the inelastic or plastic action, which is not indicated accurately by the stresses computed on the assumption of elastic behavior.

When an engineering structure is loaded in some way it will respond in a manner which depends on the type and magnitude of the load, and the strength as well as stiffness of the structure. Whether the response is considered satisfactory depends on the requirements which must be satisfied.

Steel beams transfer transverse loads by flexural action into say, column in a structural frame. Beams can be designed as simply supported or continuous (Steel Construction Institute, 1991). The effective span of the beam depends on its end conditions and type of restraint. Beam can be restrained against lateral torsional buckling either throughout its length or by the stiffeners at intermediate points (MacGinley and Ang, 1990).

The resistance of a structural member as well as the loads applied to it is a function of several variables, most of which are random (Melchers, 1999). Therefore, the use of probabilistic approach in the design of structures enables the structural safety to be treated in a more rational manner.

The study of structural reliability is concerned with the calculation and prediction of the probability of limit state violation for engineered structures at any stage during their life. In particular, the study of structural safety is concerned with the violation of the ultimate or serviceability limit states for the structure (Madsen *et al.*, 1986). Reliability is therefore the branch of structural engineering which is concerned with the analysis and probabilistic assessment of design random variables in order to predict whether specified limit state would be violated and in doing this, uncertainties inherent in structural design have to be taken into consideration (Doty, 1985).

The effect of uncertainties in design is included by the use of safety factors that are based on engineering judgment and previous experience with similar structure. Due to the fact that safety involves a consideration of random variables and the realization of the limitations in design by the deterministic method, it is now generally accepted that the rational approach to the analysis of safety is through the use of probabilistic models (Morris and Plum, 1987). Under-estimation of these uncertainties sometimes leads to adverse results of collapse such as those reported by Lew *et al.*(1982) and Igba (Igba 1996). In general, because of uncertainties, the question of safety and performance has arisen (MacGinley and Ang,, 1990).

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Hence it is necessary to evaluate the level of safety implied in ultimate and serviceability design criteria of simply supported steel beams. The BS5950 has been found to be very conservative (Abubakar and Sanusi, 2006).

The work presented investigates the safety associated with the design criteria of grade 43 simply supported steel beams considering both ultimate and serviceability limit states. The beam was assumed to be restrained against lateral and torsional buckling throughout its length.

First Order Reliability Procedure:

Probabilistic design is concerned with the probability that a structure will realize the functions assigned to it. In this work, the reliability method employed is briefly reviewed.

If R is the strength capacity and S the loading effect(s) of a structural system which are random variables, the main objective of reliability analysis of any system or component is to ensure that R is never exceeded by S. In practice, R and S are usually functions of different basic variables. In order to investigate the effect of the variables on the performance of a structural system, a limit state equation in terms of the basic design variable is required. This limit state equation is referred to as the performance or state function and expressed as:

$$g(x_i) = g(x_1, x_2, ..., x_n) = R - S \quad (1)$$

where X_i, for i = 1,2,..., n, represent the basic design variables.
The limit state of the system can then be expressed as

$$g(x_i) = 0. \quad (2)$$

Graphically, the line $g(x_i) = 0$ represents the failure surface while $g(x_i) > 0$ represents the safe region and $g(x_i) < 0$ corresponds to the failure region. This is shown in Fig. 1. Introducing the set of uncorrelated reduced variates,

$$x'_i = \frac{(X_i - \mu_{x_i})}{\sigma_x}, i = 1,2,....n \quad (3)$$

and in terms of these reduced variates the limit state equation becomes:

$$g(\sigma_{xi}X'_1 + \mu_{xi}, \sigma_{x2} X'_2 + \mu_{x2},.., \sigma_{xn}X'_n + \mu_{xn}) = 0, \quad (4)$$

where μ and s are the means and standard deviations of the design variables. The distance D, from a point $X'_i = (X'_1, X'_2, ..., X'_n)$ on the failure surface $g(x'_i) = 0$ to the origin of X_i space is also given as

$$D = \sqrt{X'^2_1 + X'^2_2 + ... + X'^2_n} \quad (5)$$

In matrix form:

$$D = (X'_1 X'_2 \cdots X'_n)\begin{vmatrix} X'_1 \\ X'_2 \\ . \\ . \\ . \\ X'_n \end{vmatrix} = (X^t_i X)^{1/2} \quad (6)$$

The point on the failure surface (X'^*_1, X'^*_2, ..., X'^*_n), having the minimum distance to the origin may be determined by minimizing the function D and subjecting equation (6) to the constraint $g(X_i) = 0$. For this purpose, the method of Langrange's multiplier may be used. Let

$$L = D + \lambda g(X_i) \tag{7}$$

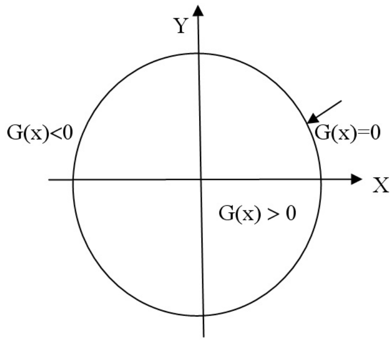

Fig.1: The most likely failure point (Thoft-Christensen and Baker, 1982)

where D is the minimum distance to the origin of the circle in Fig. 3, l is the value of the Langrange's multiplier and $g(X_i)$ is the limit state function.

Substituting equation (6) in (7) gives

$$L = (X'^{t}_{i} X'_{i})^{1/2} + \lambda g(X_i) \tag{8}$$

where λ is the value of the multiplier. In scalar notation,

$$L = \sqrt{X'^2_1 + X'^2_2 + \ldots + X'^2_n} + \lambda g(x_1, x_2, \ldots, x_n) \tag{9}$$

in which $X_i = \sigma_{xi} X'_i + \mu_{xi}$ where μ_{xi} and σ_{xi} are the means and standard deviations of the design variables. Minimizing L, we obtain (n+1) equations with (n+1) unknown as

$$\frac{\partial L}{\partial x_i} = \frac{X'_i}{\sqrt{X'^2_1 + X'^2_2 + \ldots + X'^2_n}} + \ldots + \frac{\lambda \partial g}{\partial X'_i} = 0 \tag{10}$$

and,

$$\frac{\partial L}{\partial \lambda} = g(X_1, X_2, \ldots, X_n) = 0 \tag{11}$$

The solution to equations (10) and (11) would yield the most probable failure point $(X'^*_1, X'^*_2, \ldots, X'^*_n)$. Introducing the gradient vector,

$$G = \frac{\partial g}{\partial X'_1}, \frac{\partial g}{\partial X'_2}, \ldots, \frac{\partial g}{\partial X'_n} \tag{12}$$

in which

$$\frac{\partial g}{\partial X'_i} = \frac{\partial g}{\partial X_i} \cdot \frac{\partial X_i}{\partial X'_i} = \sigma_X \frac{\partial g}{\partial X_i} \tag{13}$$

Therefore, in vector form we have

$$\frac{X'}{(X'^t X)^{1/2}} + \lambda G = 0 \tag{14}$$

From which

$$X' = \lambda D G \tag{15}$$

From equation (6)

$$D = [(\lambda D G^t)(\lambda D G)]^{1/2} = \lambda D (G^t G)^{1/2} \tag{16}$$

and,

$$\lambda = (G^t G)^{-1/2} \tag{17}$$

Where G^t is the transpose of the gradient vector G. Substituting equation (17) into equation (15) gives

$$X' = \frac{-GD}{(G^t G)^{1/2}} \tag{18}$$

Multiplying both sides of equation (18) by G^t, the transpose of the gradient vector matrix, we have

$$G^t X' = \frac{-G^t G D}{(G^t G)^{1/2}} = -(G^t G)^{1/2} D \tag{19}$$

which implies

$$D = \frac{-G^t X'}{(G^t G)^{1/2}} \tag{20}$$

The minimum distance from the origin describing the variable space to the line representing the failure surface equals β and therefore equation (20) becomes

$$\beta = \frac{-G^* X'^*}{(G^* G^*)^{1/2}} \tag{21}$$

where G* is the gradient vector at the most probable failure point (X'^*_1, X'^*_2, ..., X'^*_n). It is the value of β which tells us of the safety of any given design under uncertainties in the decision variables.

Performance Functions:

The calculation of the performance functions is performed for discrete combination of basic variables into the following equation considering bending moment condition:

$$p_y s_x - \frac{Q_k L^2 (1.4 a_o + 1.6)}{8} \tag{22}$$

and, into equations (23) and (24) considering shear and deflection failure criteria respectively:

$$0.6 p_y D t - \frac{Q_k L (1.4 a_o + 1.6)}{\text{(24)}\ 2} \tag{23}$$

$$\frac{L}{360} - \frac{5 L^4 Q_k (1.4 a_o + 1.6)}{384 EI} \tag{24}$$

In equations (22) to (24), p_y is yield strength of steel, s_x is plastic sectional modulus along the major axis of the member, Q_k is magnitude of imposed load, α_o is the load ratio, D is the overall depth of chosen section, t is the thickness of web, L is the design span, E is material elastic modulus and I is the second moment of area of the section.

Design is said to be satisfactory if conditions set-out in the code of practice is satisfied by estimating

$$P_f = P(G(X) \leq 0), \quad (25)$$

for varying values of the relevant design variables in the limit state equation.

The procedure of the FORM in the previous section, which was coded in a FORTRAN module (Gollwitzer 1988), was employed for the computation of the reliability indices.

Example of a Simply Supported Beam:

A grade 43 simply supported steel beam designed to transmit a uniformly distributed imposed load of 6 kN/m was designed in accordance with the provisions set-out in BS5950 (1990).

The beam has a clear span of 7.2m and is fully restraint against lateral torsional buckling throughout its span by a 150mm thick reinforced concrete slab. A universal beam 457 x 152 x 60 kg/m was opted for as an economical section that satisfies both the ultimate and serviceability limit states of BS5950 (1990).

Results of Reliability Analysis:

Reliability analyses of simply supported steel beam designed in the previous section was achieved by the use of FORM by estimating the reliability levels at varying values of span, L; load ratio, α_o and imposed load, Q_k. Safety indices were obtained from the programs, and plots of the safety indices versus the varied design variables were as shown in Figs. 2 to 5, considering moment failure criterion; Figs. 6 to 9 considering shear failure criterion; while Figs. 10 to 13 when deflection failure criterion was considered. From the plots it can be observed that:

a- Generally, as the span of the beam increases, the safety of the designed section decreases.
b- Also, as the load ratio increases, the safety of the designed element decreases.
c- It could also be seen that at higher load ratios and spans of 10m and beyond, in case of bending condition; 11m and beyond, in the case of shear condition and 7m and beyond, in the case of deflection condition; the negative safety indices imply that, the corresponding load ratios are not practicable (NKB, 1978).
d- Again, when the span (L) of the beam and the load ratio (α_o) are kept constant, as the magnitude of imposed load (Q_k) increased by 200%, the safety of the designed section decreased by about 66% considering bending, about 84 % considering shear and by about 82% when deflection criterion was considered.
e It was shown from Figs. 2 to 13 that the BS5950 (1990) design criteria of simply supported steel beams are fairly consistent.
f The safety indices considering deflection are conservative (NKB, 1978). Therefore, design of simply supported beams in accordance with BS5950 (1990) considering deflection failure criterion leads to uneconomical designs.

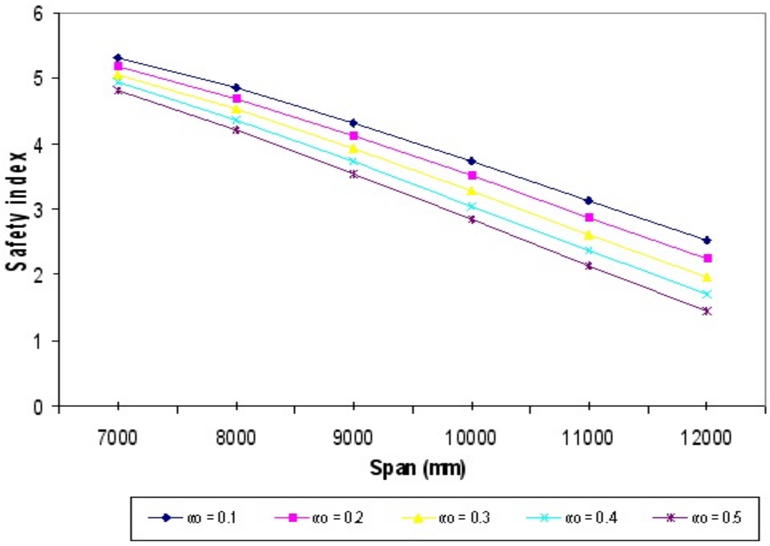

Fig. 2: Safety Index versus Span, Imposed load of 6kN/m (Bending Condition).

g The weighted average safety indices are respectively 2.03, 5.69 and 1.72 for bending, shear and deflection failure criteria of the BS5950 (1990).

h- Based on (g) above, BS5950 design (Vrouwenvelder, 2000) results seems conservative with respect to shear, unsafe with respect to bending (under low shear load), and satisfactory with respect to deflection.

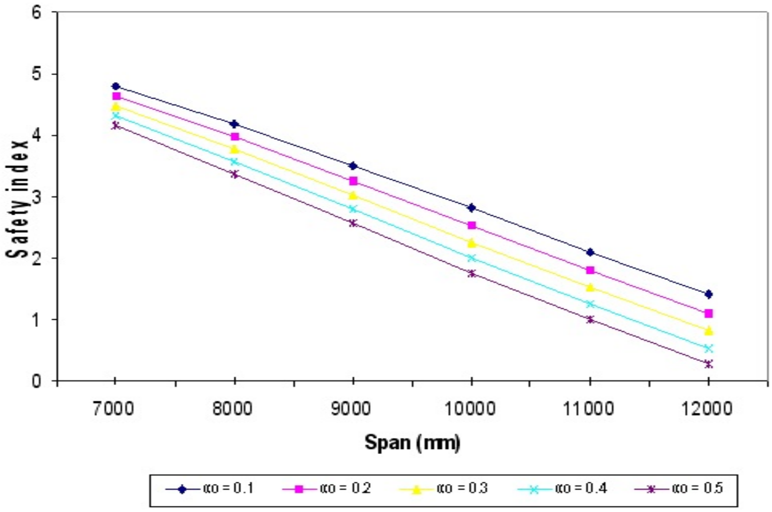

Fig. 3: Safety Index versus Span, Imposed load of 8kN/m(Bending Condition).

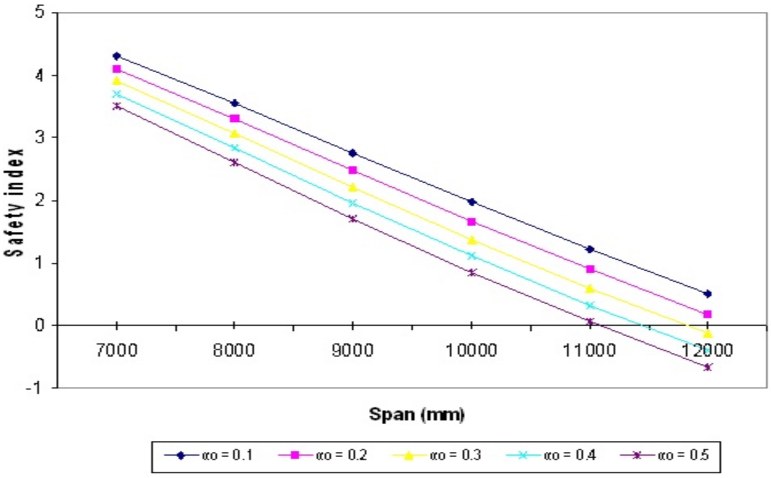

Fig. 4: Safety Index versus Span, Imposed load of 10kN/m (Bending Condition).

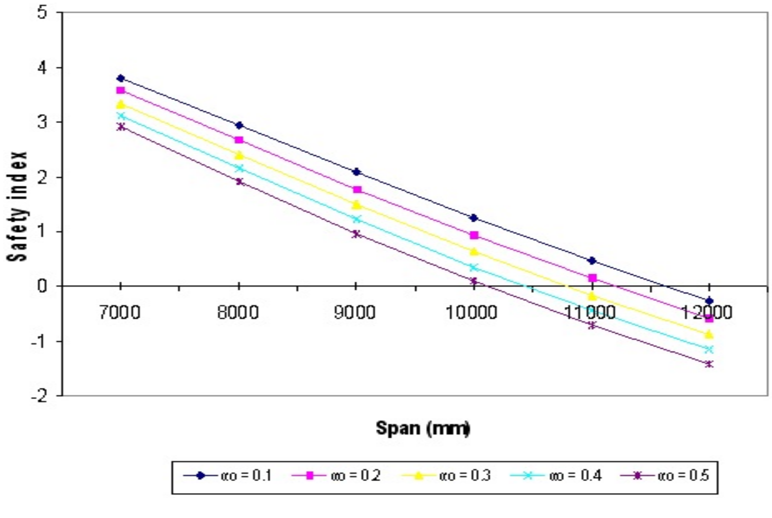

Fig. 5: Safety Index versus Span, Imposed load of 12kN/m (Bending Condition).

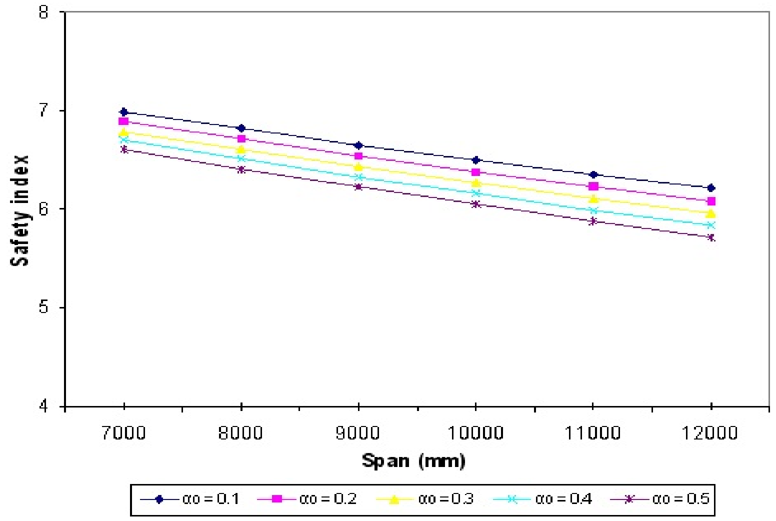

Fig. 6: Safety Index versus Span, Imposed load of 6kN/m (Shear Condition).

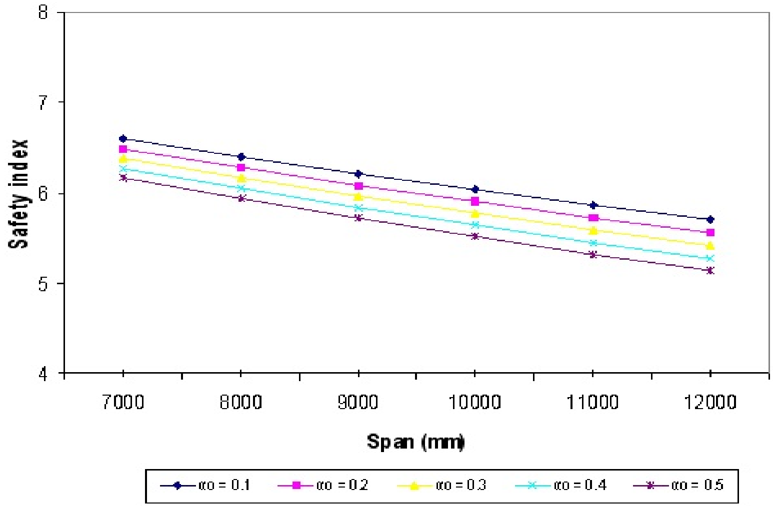

Fig. 7: Safety Index versus Span, Imposed load of 8kN/m (Shear Condition).

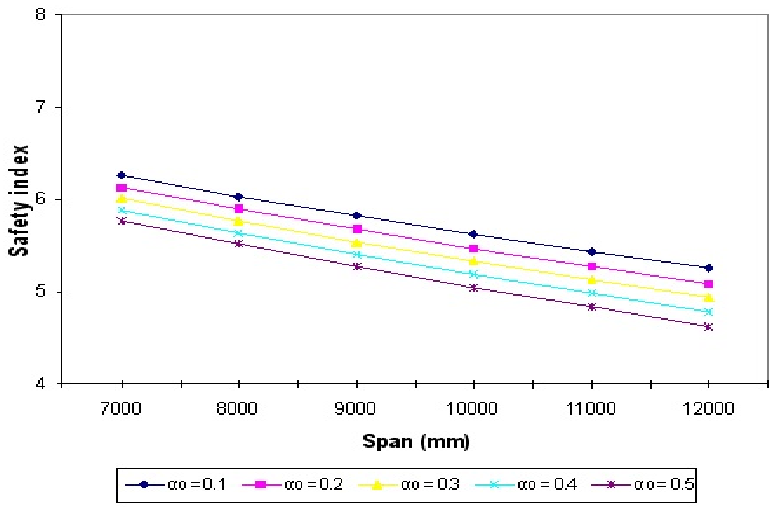

Fig. 8: Safety Index versus Span, Imposed load of 10kN/m (Shear Condition).

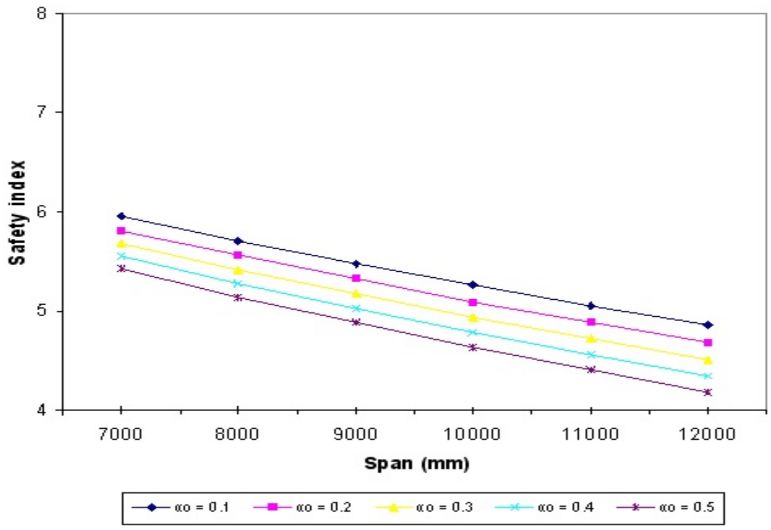

Fig. 9: Safety Index versus Span, Imposed load of 12kN/m (Shear Condition).

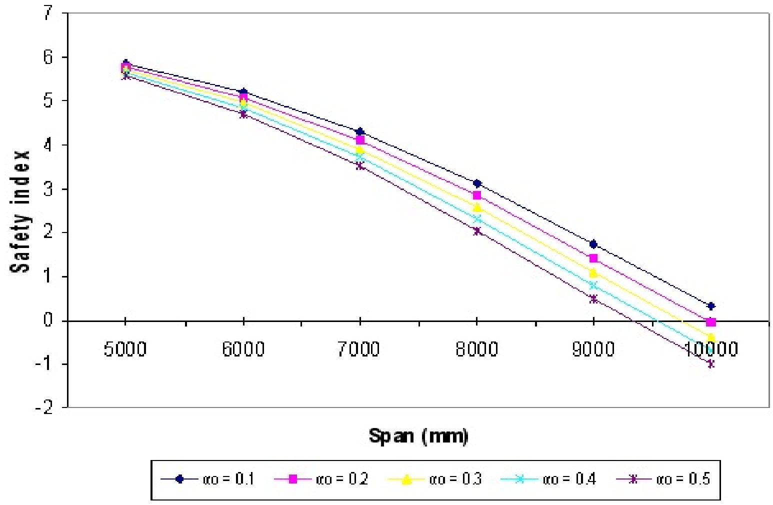

Fig. 10: Safety Index versus Span, Imposed load of 6kN/m (Deflection Condition).

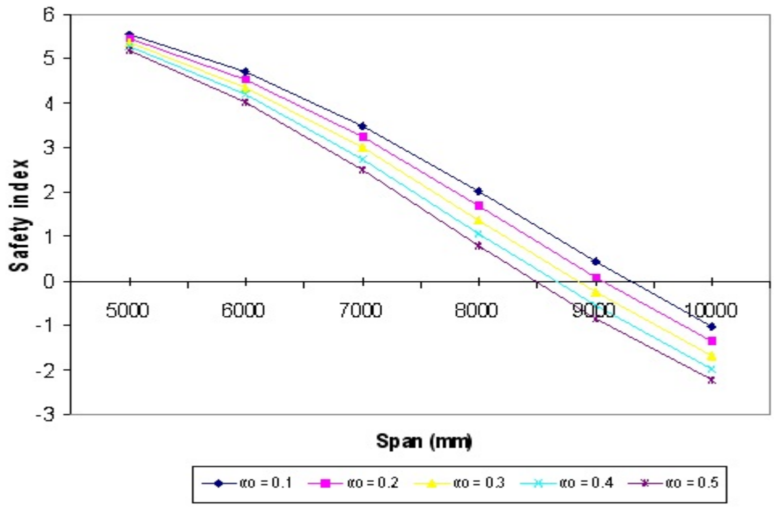

Fig. 11: Safety Index versus Span, Imposed load of 8kN/m (Deflection Condition).

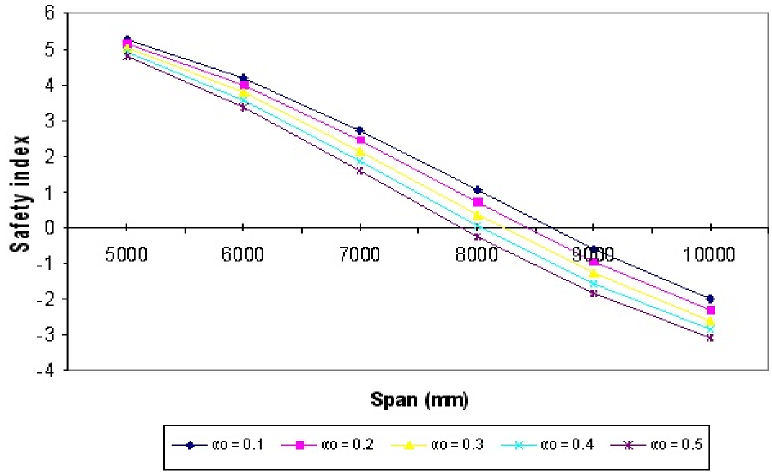

Fig. 12: Safety Index versus Span, Imposed load of 10kN/m (Deflection Condition).

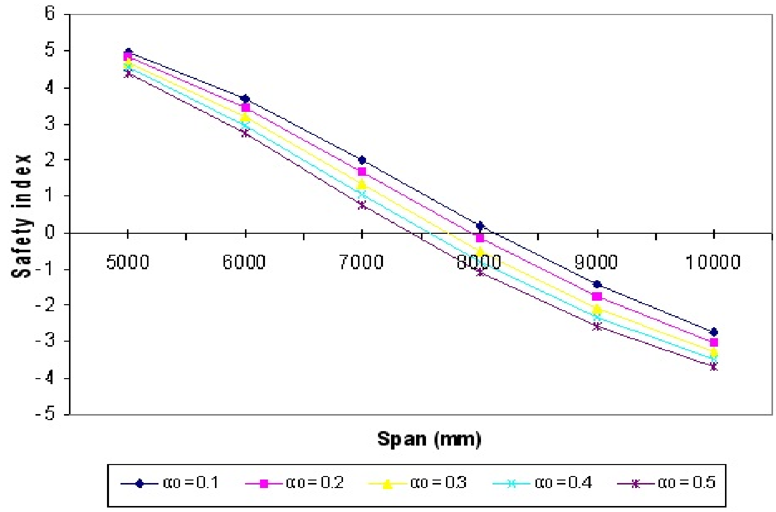

Fig. 13: Safety Index versus Span, Imposed load of 12kN/m (Deflection Condition).

Conclusion:

Reliability analysis of simply supported steel beam considering both ultimate and serviceability limit states was investigated using FORM. The results of investigation showed that the BS5950 design procedure of the beam is fairly consistent. Also, when the span (L) of the beam and the load ratio (α_o) are kept constant, as the magnitude of imposed load (Q_k) increased by 200%, the safety of the designed section decreased by 64% considering bending, 84% considering shear and by 82% when deflection criterion was considered. Also, the weighted average safety indices are respectively 2.03, 5.69 and 1.72 for bending, shear and deflection failure criteria of the BS5950 (1990). Therefore, BS5950 design results seems conservative with respect to shear, unsafe with respect to bending (under low shear load), and satisfactory with respect to deflection.

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