Australian Journal of Basic and Applied Sciences, 2(4): 1204-1208, 2008
ISSN 1991-8178

Sum of Linear and Fractional Multiobjective Programming Problem under Fuzzy Rules Constraints

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Abstract: This paper deals with the solution procedure of the sum of linear and fractional multiobjective programming problem in which the fractional relationship between the decision variables and the objective functions is not completely known. Our knowledge base consists of a block of fuzzy If-then rules, where antecedent part of the rules contains some linguistic value of the decision variables and the consequence part is linear combination of the crisp value of the decision variable. We suggest the use of Takagi and Sugeno fuzzy reasoning method to determine the crisp functional relationship between the objective function and the decision variables under the assumption that the denominator of the fractional part of the objective functions is non-zero on the constraints set and finally solving the resulting programming problem to find a fair optimal solution of the original problem.

Key words: Multiobjective programming, Linear plus fractional programming, If-then rules, Fuzzy reasoning method.

INTRODUCTION

Fuzzy programming problem can be stated and solved in many different ways as given by Zimmermann (1975) and (1992). Earlier researchers, considers programming problem of the form

$$\begin{aligned} &\text{Max / Min.} && f(x) \\ &\text{Subject to,} && x \in X \end{aligned}$$

where f or / and X are defined by fuzzy terms. After that searching for a crisp x^* which maximizes/ minimizes f on X. Similarly fuzzy linear programming problems (FLP) can be stated as given by Herrera (1992), Kovacs (1991) and Rommelfanger (1996) as

$$\begin{aligned} &\text{Max./Min.} && f(x) = \hat{C}x \\ &\text{subject to,} && \tilde{A}x \le \tilde{b} \end{aligned} \qquad (1)$$

where the fuzzy terms are denoted by tilde.

Carlsson and Fuller (2001) have considered constrained fuzzy optimization problem with single objective of the form

$$\begin{aligned} &\textbf{Max./Min.} && f(x) \\ &\textbf{Subject to,} && \{R_1(x), R_2(x), \ldots\ldots, R_m(x) \mid x \in X \subset R^n\} \end{aligned} \qquad (2)$$

with R_i (x): if x_1 A_{i1} and,............,and x_n is A_{in} then f(x) is c_i where A_{ij} an c_i are fuzzy numbers and they have suggested the use of Tsukemoto's fuzzy reasoning method given by Tuskamoto (1979) to determine the crisp value of f.

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Later DadashZadeh and Nimse (2005) extended the principle for multiple objective optimization problem under fuzzy If-then rules (1998). and considered the multiple objective optimization problem of the form

$$\text{Max./Min. } \{ f_1(\mathrm{x}), f_2(\mathrm{x}), \ldots\ldots, f_k(\mathrm{x}) \} \tag{3}$$

where $\mathrm{x}_1, \mathrm{x}_2, \ldots\ldots, \mathrm{x}_n$ are linguistic variables and

$R_i(\mathrm{x})$: if x_1 is A_{i1} and, ..., and x_n is A_{in} then

$$f_1(\mathrm{x}) = \sum_{j=1}^{n} a_{ij}^1 x_j + b^1, \ldots\ldots, f_k(\mathrm{x}) = \sum_{j=1}^{n} a_{ij}^k x_j + b^k, \quad i = 1, 2, \ldots\ldots, m$$

$$\mathrm{x} \in X$$

Such that $\mathrm{X} = \mathrm{X}_1 \times \mathrm{X}_2 \times \ldots\ldots \times \mathrm{X}_n$ and $\mathrm{x} = (x_1, x_2, \ldots, x_n)$
about the value of $f_1, f_2, \ldots\ldots, f_k$ and A_{ij} is a fuzzy number and a_{ij} and b_i are real numbers.
determine the crisp value of the k-th objective function f_k at $u \in R^n$ by the fuzzy reasoning method given by Takagi and Sugeno (1985) and obtain an optimal solution of (3) by solving the resulting multiobjective non-linear mathematical programming problem without constraints

$$\text{Max./Min.} \quad \{ f_1(u), f_2(u), \ldots\ldots, f_k(u) \}$$
$$\text{subject to,} \quad u \in X.$$

The present paper is organized as follows: In section 2, we describe the problem and notations. In section 3, we propose the solution procedure for multiobjective linear plus fractional programming problem under fuzzy If-then rules and related definitions and properties. An example is given in last to support our proposed model.

The Problem:
We consider the multiobjective linear plus fractional programming problem of the form

$$\text{Max./Min.} \{ f_1(\mathrm{x}), f_2(\mathrm{x}), \ldots\ldots, f_k(\mathrm{x}) \}$$

where $\mathrm{x} = (x_1, x_2, \ldots, x_n)$ are linguistic variables and

$R_i(\mathrm{x})$: if x_1 is A_{i1} and, ..., and x_n is A_{in}, then

$$f_1(\mathrm{x}) = (L_{i1}\mathrm{x} + l_{i1}) + \left(\frac{C_{i1}\mathrm{x} + c_{i1}}{D_{i1} + d_{i1}} \right),$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$

$$f_k(\mathrm{x}) = (L_{ik}\mathrm{x} + l_{ik}) + \left(\frac{C_{ik}\mathrm{x} + c_{ik}}{D_{ik} + d_{ik}} \right), \quad i = 1, 2, \ldots\ldots, m$$

$$\mathrm{x} \in X$$

Such that $\mathrm{X} = \mathrm{X}_1 \times \mathrm{X}_2 \times \ldots\ldots \times \mathrm{X}_n$ and $\mathrm{x} = (x_1, x_2, \ldots, x_n)$
about the value of $f_1, f_2, \ldots\ldots, f_k$ and A_{ij} is a fuzzy number and L_{ij}, C_{ij},

$i = 1, 2, \ldots\ldots, m$ are real vectors and l_{ij}, c_{ij}, d_{ij} are real number. Here we restrict our self for only the values for which denominator of the fractional part of the objective functions is non-zero. Finally we determine the crisp value of the k-th objective function f_k at $u \in R^n$ by the fuzzy reasoning method given by Takagi and Sugeno (1985) and this reduces the problem into multiobjective fractional programming problem without constraints in the form

$$i = 1, 2, \ldots, m, \quad \text{Max./Min.} \quad \{f_1(u), f_2(u), \ldots\ldots, f_k(u)\}$$
$$\text{subject to,} \quad u \in X.$$

This problem can be solved using fuzzy programming method given by Gupta and Chakraborty (1997) and a compromise optimal solution of problem Mark eq. no. can be obtained.

Multiobjective Linear plus Fractional Programming Problem under Fuzzy If-then Rules:

As per Zadeh (1975), linguistic variable can be regarded either as a variable whose value is a fuzzy number or as a variable whose value is defined in linguistic terms.

Definition:

A t-norm T is a function T:[0,1]×[0,1]→[0,1] having the following four properties.

$$(i)\ T(x,1) = x, \quad \forall x \in [0,1]$$
$$(ii)\ T(x,y) = T(y,x)$$
$$(iii)\ T(x,T(y,z)) = T(T(x,y),z) \quad \forall x,y,z \in [0,1]$$
$$(iv)\ T(x,y) \le T(x^1,y^1), \quad if\ x \le x^1 \text{ and } y \le y^1$$

For any t-norm T with use of property (iii), we have

$$T_0(x) = x,$$
$$T_n(x_1,x_2,\ldots,x_n) = T(T_{n-1}(x_1,x_2,\ldots,x_{n-1}),x_n), \quad n \ge 2$$

For obtaining a fair optimal solution to the fuzzy optimization problem

$$\text{Max./Min.}\,\{f_1(x), f_2(x), \ldots\ldots, f_k(x)\}$$

Subject to, $\{R_1(x), R_2(x), \ldots\ldots, R_m(x) \mid x \in X\}$ (5)

with fuzzy If-then rules of form (4), we determine the crisp value of the k-th objective function f_k at $u \in R^n$ from the fuzzy rule base R using the fuzzy reasoning method given by Takagi and Sugeno (1985) as

$$F_s(u) = \frac{\alpha_1 Z_{1s}(u) + \ldots\ldots + \alpha_m Z_{ms}(u)}{\alpha_1 + \alpha_2 + \ldots\ldots + \alpha_m}, \quad s = 1, 2, \ldots\ldots, k$$

Where the firing levels of the rules are computed by

$$\alpha_i = T(A_{i1}(u_1), A_{i2}(u_2), \ldots\ldots\ldots, A_{im}(u_m)), \quad i = 1, 2, \ldots\ldots, m$$

And the individual rule outputs denoted by $Z_{is}(u)$ are divided from the relationship

$$Z_{is}(\mathrm{u}) = (L_{is}\mathrm{x} + l_{is}) + \left(\frac{C_{is}\mathrm{x} + c_{is}}{D_{is} + d_{is}}\right), \qquad i = 1, 2, \ldots\ldots, m$$

Here $\mathrm{u} = (u_1, u_2, \ldots\ldots, u_n)$ is an input vector and to determine the firing level of the rules, we suggest the use of the product t-norm. In this way our constrained multiobjective optimization problem (5) turns into the following crisp unconstrained multiobjective fractional programming problem.

$$\begin{array}{ll} \text{Max./Min.} & \{f_1(u), f_2(u), \ldots\ldots, f_k(u)\} \\ \text{subject to,} & u \in X. \end{array} \tag{6}$$

Example:
Consider the sum of linear and fractional multiobjective optimization problem

$$\text{Max.}\{f_1(\mathrm{x}), f_2(\mathrm{x})\}$$

Subject to, $R_1(x)$: if x_1 is A_1 and x_2 is A_2, then

$$f_1(x_1, x_2) = (-x_1 + 1) + \frac{x_1}{x_1 - x_2 + 1}, \text{ and } f_2(x_1, x_2) = (x_1 - 1) + \frac{x_1 + x_2}{x_1 + 1}$$

$R_2(x)$: if x_1 is A_2 and x_2 is A_1, then

$$f_1(x_1, x_2) = -x_1 + \frac{x_1 + x_2}{x_1 - x_2 + 1}, \text{ and } f_2(x_1, x_2) = x_1 + \frac{x_1}{x_1 + 1}$$

$\forall\ x_1, x_2 \in [0, 1]$, such that A_1 and A_2 are fuzzy subsets in $[0, 1]$ with membership functions $\mathrm{A}_1(\mathrm{x}) = \mathrm{x}$, and $\mathrm{A}_2(\mathrm{x}) = 1 - \mathrm{x}$

Suppose that $u = (u_1, u_2) \in X$ is an input vector.

$$\alpha_1 = T(A_1(u_1), A_1(u_2)) = A_1(u_1).A_1(u_2) = u_1 u_2$$
$$\alpha_2 = T(A_2(u_1), A_1(u_2)) = A_2(u_1).A_1(u_2) = (1 - u_1)u_2$$

where T is a product t-norm.

$$Z_{11} = (-u_1 + 1) + \frac{u_1}{u_1 - u_2 + 1}, \qquad Z_{12} = (u_1 - 1) + \frac{u_1 + u_2}{u_1 + 1}$$

$$Z_{21} = -u_1 + \frac{u_1 + u_2}{u_1 - u_2 + 1}, \qquad Z_{22} = u_1 + \frac{u_1}{u_1 + 1}$$

$$f_1(u) = \frac{\alpha_1 Z_{11}(u) + \alpha_2 Z_{21}(u)}{\alpha_1 + \alpha_2} = \frac{u_1 + u_2 - u_1 u_2}{u_1 - u_2 + 1}$$
$$f_2(u) = \frac{\alpha_1 Z_{12}(u) + \alpha_2 Z_{22}(u)}{\alpha_1 + \alpha_2} = \frac{u_1 u_2 - u_1}{u_1 + 1}$$

Now, the reduced problem

$$\text{Max.}\left\{\frac{u_1+u_2-u_1u_2}{u_1-u_2+1}, \frac{u_1u_2-u_1}{u_1+1}\right\},$$

subject to, $u_1, u_2 \in [0,1]$

Using non-linear fuzzy programming techniques as given by Gupta and Chakraborty (1997) the compromise optimal solution of this problem be $u_1 = u_2 = 1$.

REFERENCES

Carlsson, C. and R. Fuller, 2001. Optimization under fuzzy if then rules, *Fuzzy sets and systems*, 119: 111-120.

Carlsson, C. and R. Fuller, 1998. Multiobjective Optimization with Linguistic Variables (in Proceeding of the Sixth European Congress on Intelligent Techniques and Soft Computing (EUFIT, 98), Aachen, September 7-10, 1998), Verlag Mainz, Aachen, II: 1038-1042.

Dadashzadeh, R. and S.B. Nimse, 2005. Multiobjective Optimization under Fuzzy rule constraints, Rajasthan Academy of Physical Sciences, 4(3): 223-228.

Fuller, R. and H.J. Zimmerman, 1993. Fuzzy reasoning for solving fuzzy mathematical programming problems, Fuzzy sets and systems, 60: 121-133.

Herrera, F., M. Kovacs and J.L Verdegay, 1992. Fuzzy Linear Programming Problems with Homogeneous Linear Fuzzy Functions (in Proc. Of IPMU 1992), Universitat de les illes Balears, 361-364.

Kovacs, M. 1991, Linear Programming with cetered fuzzy numbers, Annales, Univ. Sci. Budapest (Sectio Computatorica), 12: 159-165.

Gupta, S. and M. Chakraborty, 1997. Multi objective linear programming: A fuzzy programming approach, International journal of management and system, 13(2): 207-214.

Rommelfanger, H., 1996. Fuzzy Linear Programming and Applications, European Journal of Operational Research, 92: 512-527.

Takagi, T. and M. Sujgeno, 1985. Fuzzy Identification of Systems and its applications to modeling nd control, IEEE Trans. Systems, MAN Cybernet, 116-132.

Tuskamoto, Y., 1979. An approach to Fuzzy reasoning method (in M.M. Gupta, R.K. Ragode and R.R. Yager (eds.)- Advances in Fuzzy Set Theory and Applications, North-Holland, New York).

Zadeh, L.A., 1975. The concept of Linguistic Variable and its applications to approximate reasoning, Pts. I, II, III, Information Sciences, 8: 199-251; 8: 301-357; 9: 43-80.

Zimmerman, H.J., 1975. Description and optimization of fuzzy systems internet, J.General Systems 2: 209-215.

Zimmerman, H.J., 1992. Fuzzy mathematical programming (in Struct C.Shapir (ed.) - Encyclopedia of Artificial Intelligence, John Willey and Sons, New York), 521-528.