Australian Journal of Basic and Applied Sciences, 2(1): 125-128, 2008
ISSN 1991-8178

Axisymmetric Stability of Annular Fluid Ambient with Different Liquid

[1]Ahmed E. Radwan, [2]Samia S. Elazab, [3]Kh. S. Mekheimer and [4]Shimaa L. Azwz,

[1]Mathematics Department, Faculty of Science, Ain Shams University, Cairo, Egypt.
[2]Mathematics Department, Women's University College, Ain Shams University Cairo, Egypt.
[3]Mathematics Department, Faculty of Science, Al-Azhar University,
Nasr City, 11884, Cairo, Egypt.
[4]Basic Science Department , Ministry of Higher Education, Modern Academy
for Engineering and technology, Maadi, Cairo, Egypt.

Abstract: The stability criterion for axisymmetric perturbation is derived in its general form and discussed. The model is stable in the axisymmetric mode $m = 0$ for certain wave lengths. The densities ratio is stabilizing or destabilizing according to restrictions. The electromagnetic force due to the magnetic field pervaded in the liquid region has strong stabilizing effect for all short and long wavelengths. The discussions of this relation showed that the model is capillary unstable in the axisymmetric mode $m = 0$ for small range of wave numbers and could be suppressed due to the electromagnetic force influence.

Keywords: Magnetohydrodynamic stability, hydrodynamic stability

INTRODUCTION

The stability of a full liquid jet has been discussed since many years ago cf. Rayleigh (1945). Chandrasekhar (1981) discussed the hydrodynamic and magnetohydrodynamic stability of different models. A lot of works have been carried out concerning such studies in the last decades by Radwan *et. al.* (1991, 1994 and 2007). Here we study the axisymmetric stability of a liquid density ρ surrounding a solid cylinder and that model is ambient with a different liquid of density $\rho`$ under the effect of capillary and Lorentz forces. Such study has practical applications in several domains of science.

Formulation of the Problem:

We consider a streaming (velocity $\underline{u}_0$) liquid cylinder of density ρ (of radius R_0) surrounding a solid cylinder (of radius qR_0) and that model is ambient with a streaming (velocity $\underline{u}_0$) different liquid of density $\rho`$ under the effect of the capillary and electromagnetic forces.

The basic equations which are the combination of the hydrodynamic and Maxwell's electromagnetic equations:

$$\rho \frac{d\underline{u}}{dt} = \underline{j} \wedge \underline{H} - \nabla p, \frac{d}{dt} = \frac{\partial}{\partial t} + (\underline{u}.\nabla) \tag{1}$$

$$\nabla.\underline{u} = 0 \tag{2}$$

$$\nabla.\underline{H} = 0, j = \mu(\nabla \wedge \underline{H}) \tag{3}$$

$$\frac{\partial \underline{H}}{\partial t} = (\underline{H}.\nabla)\underline{u} - (\underline{u}.\nabla)\underline{H} - \underline{H}(\nabla.\underline{u}) + \underline{u}(\nabla.\underline{H}) \tag{4}$$

$$p_s = T\nabla.\underline{N} \tag{5}$$

Corresponding Author: Ahmed E. Radwan, Mathematics Department, Faculty of Science, Ain Shams University, Cairo, Egypt.

$$\underline{N} = \frac{\nabla F(r,\varphi,z;t)}{\left|\nabla F(r,\varphi,z;t)\right|} \tag{6}$$

$$F(r,\varphi,z;t) = 0 \tag{7}$$

$$\underline{u}_o = (0,0,U) \tag{8}$$

Here $\underline{u}$ and p are the fluid velocity vector and kinetic pressure, $\underline{H}$ is the magnetic field intensity. Equation (1) is the magnetodynamic vector equation of motion including the magnetodynamic (Lorentz) force $\underline{j}\wedge\underline{H}$ and the gradient pressure force ∇p. Equation (2) is the continuity equation expressing the conservation of mass. Equations (3) and (4) are the equations of the magnetic field in the liquid region. Equation (5) gives the pressure due to the capillary force. Equation (6) is the unit normal vector $\underline{N}$ to the fluid-liquid interface and equation (7) is the equation of the boundary surface .

Unperturbed State:

The unperturbed stationary state is studied upon considering the basic equations system (1) - (7). Equation (1) reduces to

$$\nabla\Pi_o = 0, \Pi_o = p_o + \frac{\mu}{2}(\underline{H}_o.\underline{H}_o) \tag{9}$$

from which

$$p_o + \frac{\mu}{2}H_o^2 = c_1 \tag{10}$$

and

$$\Pi_o^{'} = p_o^{'} + \frac{\mu^{'}}{2}(\underline{H}_o.\underline{H}_o) = c_2 \tag{11}$$

To identify the constants of integration c_1 and c_2 we have to apply the balance of the pressure across the boundary surface at r = R_0. Taking into account equation (5) in the initial state, equation (11) gives

$$p_o = \frac{T}{R_o} - \frac{\mu}{2}(\underline{H}_o.\underline{H}_o), p_o^{'} = c_2 - \frac{\mu}{2}H_o^2 \tag{12}$$

Perturbation Analysis:

For small perturbation of the initial state, the variable quantity $Q(r,o,z,t)$ is assumed to be

$$Q(r,o,z,t) = Q_o(r) + \varepsilon(t)Q_1(r,o,z) + \ldots.. \tag{13}$$

where the subscript 0 as usual characterizes the initial quantities while those with index unity are their increments. The amplitude of the perturbation ε(t) at time t is ($\varepsilon = \varepsilon_0$ at $t = 0$) assumed to be

$$\varepsilon(t) = \varepsilon_o \exp(\sigma t) \tag{14}$$

where σ is the growth rate or rather the oscillation frequency ω as $\sigma (= i\omega \; with \; i = \sqrt{-1}$ is the imaginary factor) is imaginary.

Based on the linearized perturbation technique, the perturbed radical distance of the fluid in the axisymmetric perturbation is given by

$$r = R_o + \varepsilon_o R_o \exp(\sigma t + i(kz)) \tag{15}$$

where k (real) is the longitudinal wavenumber. The second term in the right side of equation (15) is the elevation of the surface wave measured from the initial unperturbed position.

By using the expansion (13) for equations (1)- (7), the perturbation equations are obtained in the form:

$$\frac{\partial \underline{u}_1}{\partial t} + (u_o.\nabla)\underline{u}_1 - \frac{\mu}{\rho}(\underline{H}_o.\nabla)\underline{H}_1 = -\nabla\Pi_1 \tag{16}$$

$$\rho\Pi_1 = p_1 + \mu(\underline{H}_o.\underline{H}_1) \tag{17}$$

$$\nabla.\underline{u}_1 = 0 \tag{18}$$

$$\nabla.\underline{H}_1 = 0 \tag{19}$$

$$\frac{\partial \underline{H}_1}{\partial t} = (\underline{H}_o.\nabla)\underline{u}_1 - (\underline{u}_o.\nabla)\underline{H}_1 \tag{20}$$

Similar system of equations could be obtained for the exterior liquid of density ρ`.

From the point of view of the space- time dependence (15), every small increment quantity $Q_1(r,o,z;t)$ may be expressed as:

$$Q_1(r,o,z;t) = \varepsilon_o Q_1(r)\exp[\sigma t + i(kz)] \tag{21}$$

By the use of the expansion (21), the linearized equations (16)-(20) are integrated and solved. Apart from the infinite solutions, the non-singular solutions are given as

$$\underline{H}_1 = \frac{ikH_o}{(\sigma + ikU)}\underline{u}_1 \tag{22}$$

$$\underline{u}_1 = \frac{-(\sigma + ikU)}{((\sigma + ikU)^2 + \Omega_A^2)}\nabla\Pi_1 \tag{23}$$

$$\underline{u}_1^{'} = \frac{-(\sigma + ikU^{'})}{((\sigma + ikU^{'})^2 + \Omega_A^{'2}}\nabla\Pi_1^{'} \tag{24}$$

$$\Omega_A^2 = \frac{\mu H_o^2 k^2}{\rho} \; and \; \Omega_A^{'2} = \frac{\mu H_o^2 k^2}{\rho^{'}} \tag{25}$$

$$\Pi_1 = (A_1 I_o(kr) + B_1 K_o(kr))\exp(i(kz) + \sigma t) \tag{26}$$

$$\Pi_1^{'} = B_2 K_o(kr)\exp(i(kz) + \sigma t) \tag{27}$$

where $I_o(kr)$ and $K_o(kr)$ are the modified Bessel functions of the first and second kind of order zero while A_1, B_1, B_2 are constants of integration to be determined upon appropriate boundary conditions

Boundary Conditions:

The solution of equations (1)-(7) in the unperturbed and perturbed states must satisfy the boundary conditions.

- The normal component of the velocity of annular liquid must be compatible with the velocity of the surrounding liquid at $r = R_o$ i.e. $u_{1r} = u'_{1r}$.
- The radial component of the velocity vector must vanish across the cylindrical solid edge at $r = qR_o$ i.e. $u_{1r} = 0$.
- The normal components of the magnetic field must be continuous across the fluids interface at $r = R_o$.

Moreover, apply some compatibility condition that the normal component of the total stresses must be continuous across the fluids interface at $r = R_o$ i.e. $\rho'\Pi'_1 + p_{1s} = \rho\Pi_1$, from which we get:

$$(\sigma + ikU)^2 = \frac{-T}{\rho R_o^3}(1-x^2)\frac{xL^o_{x,y}}{K'_o L^o_y - sL^o_{x,y}} + \frac{\mu H_o^2 x^2}{\rho R_o^2}\left(\frac{K_o L^o_{x,y} - L^o_y}{K'_o L^o_y - sL^o_{x,y}}\right) \tag{28}$$

where $x = (kR_o)$ is the dimensionless longitudinal wave number and $L^o_y = K'_o(y)I_o(x) - I'_o(y)K_o(x)$, $L^o_{x,y} = K'_o(x)I'_o(y) - K'_o(y)I'_o(x)$

We may recover some results have been already published previously upon considering appropriate simplification of the parameters of the model, cf. Radwan (1991, 1994 and 2007) and also Chandrasekhar (1981).

The dispersion relation (28) have been discussed analytically and numerically and we found the following results.

- The electromagnetic force has strong stabilizing effect for all short and long wavelengths.
- The influence of capillary force is unstable in the axisymmetric mode for certain wave lengths.
- The effect of densities ratio is stabilizing or destabilizing according to restrictions.
- The streaming has strong destabilizing effect.

REFERENCES

Abramowitz, M. and I. Stegun, 1970. “Handbook of Mathematical Functions “ (Dover publ., New York, U.S.A).

Chandrasekhar, S. and E. Fermi, 1953. Astrophys J., 118: 116.

Chandrasekhar, S., 1981. “Hydrodynamic and Hydromagnetic Stability” (Dover publ., New York, U.S.A).

Gradshteyn, I. and I. Ryzhik, 1965. “Tables of Integrals Series and products” (Academic Press, London,U.K.)

Ogansian S., 1956. Z Astronom, 33: 928.

Radwan, A.E and S Elazab, 1988. J. Magn. Magn. Matrs. (USA), 72: 219.

Radwan, A.E and S. Elazab, 1989. Astrophys. Space Sci. (UK), 158: 28.

Radwan, A.E, 1991. J. Magn. Magn. Matrs, 98: 162.

Radwan, A.E . *et al.,* 1994. Physica Scripta, (Sweden), 50: 142.

Radwan, A.E., 2007. Kyngpook Math. J. (Korea), 47: 455.

Roberts, P.H., 1967. An Introduction of MHD (Longman, London U.K)