

## A Mixed-integer Programming Model to Determine the Optimum Design of Flexible Pavement Structure

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**Abstract:** To determine the optimum structure and thickness of each layer of a pavement based on the AASHTO flexible pavement design method, a mixed-integer programming model has been developed in this paper. Using this optimization model, a unique thickness for each layer—hence the lowest cost for the construction of a pavement – is obtained. In this model, first the pavement is assumed to be a 5-layer system consisting of wearing course, binder course, black base, granular base and granular subbase and then, the optimum structure and layers thicknesses are determined. Considering the strength of subgrade, traffic volume and cost of materials and construction, it is possible to eliminate any of the black base, granular base or the granular subbase layers in the optimum structure. Under present conditions (high cost of materials in Iran), the use of the black base is not economical; neither is the construction of the granular subbase when the strength of the subgrade is high. Moreover, with a decrease in the traffic volume, and at lower strengths of subgrade, there is no need for the construction of the subbase layer. Using this model and the yearly price-list of materials and construction, it is possible every year, to draw some charts to determine the optimum thickness of different layers of pavements under various design parameters and conditions. Using these charts, it is possible to determine the optimum structure and thickness of the layers.

**Key words:** Optimum Pavement design, Mixed-Integer programming, AASHTO Method

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### INTRODUCTION

Flexible pavement design, using AASHTO method, is based on studies and tests that AASHTO carried out in Ottawa and Illinois between the years 1958 and 1960. The first AASHTO pavement design Guide was published in 1961 and revised in 1972 and 1981. Again, during the years 1984 and 1985, a committee, consisting of AASHTO experts and some consultant engineers, revised it under NCHRP 20-7/24 Project and, after some modifications, presented the 1986 AASHTO pavement design manual (Huang, 1993). Then in 1993 the method was revised again and led to the publication of the 1993 version of the manual.

The concept of logical pavement design was first introduced in the 1960s (Monismith and *et al.*, 1961; Dorman, 1962). The developed version of this method, known as “mechanistic-empirical method”, has been adopted and is used nowadays by many countries and institutions for the design of flexible pavements (Austroad, 2004; NCHRP, 2008; IRC:37, 2001; Shell, 1978; Theyse and *et al.*, 1996; Asphalt Institute, 1999; LCPC and SETRA, 1997). However, due to the need for equipped labs and, because extensive research is still needed for the use of mechanistic-empirical method, only empirical methods are used for pavement design in many countries (AASHTO, 1993; Japan Road Association, 1989; RN-29, 1970; RstO 2000, 1999).

In Iran, flexible pavement design method is based on that of the 1993 version of AASHTO manual (TASB, 2002). Pavement design, using this method, has led to some linear inequalities all of which must be satisfied simultaneously. The first optimization model for the determination of optimum structure and thickness of pavement layers, based on the method proposed in "Iran highway asphaltic pavements code" was presented by Fakhri and Ghanizadeh. The model, in the form of a linear programming model, could determine the optimum structure and thickness of pavement layers. It could only determine the optimum structure of pavements consisting of asphalt, granular base and granular subbase layers. This model was unable to determine the construction thickness of asphalt materials consisting of wearing course, binder course and black base, based on nominal maximum aggregate size of granular materials in each layer (Fakhri and Ghanizadeh, 2004).

Results obtained from the solution of this model show the fact that the optimum structure and thickness of a pavement are directly related to the ratio of the construction cost to the layer coefficient of each layer.

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The lower is the above ratio for a layer the thicker should that layer in the pavement structure become so that the pavement may be economical (Fakhri and Ghanizadeh, 2004).

In this paper a mixed-integer linear programming model has been developed to determine the optimum structure of pavement and thickness of each layer, based on minimizing the cost of pavement construction.

### MATERIALS AND METHODS

Equation (1) shows the basic equation of the design of flexible pavements using AASHTO method.

$$\log W_{8.2} = Z_R S_0 + 9.36 \log(SN + 1) - 0.2 + \frac{\log \left( \frac{\Delta PSI}{4.2 - 1.5} \right)}{0.4 + \frac{1094}{(SN + 1)^{5.19}}} - 2.3 \log \left( \frac{M_R}{0.069} \right) - 8.07 \quad (1)$$

Where

$W_{8.2}$  = Predicted number of 80-KN single axel load applications.

$Z_R$  = standard normal deviate.

$S_0$  = Combined standard error of traffic prediction and performance prediction.

$\Delta PSI$  = Difference between initial design serviceability index,  $P_0$  and terminal design serviceability index,  $P_t$ .

$M_R$  = Resilient modulus(kg/cm<sup>2</sup>)

In this equation all the parameters are known except the pavement structural number (SN). So, it is possible to find the value of SN by solving equation (1) using iteration method. It can also be solved with the help of existing graphs.

After SN is found, it is possible to find the thickness of each layer by converting SN to real thickness of the constituent layers. The thickness of each layer should be so found that the following equation can be totally satisfied.

$$SN = \frac{1}{2.54} \sum_{i=1}^n m_i a_i d_i \quad (2)$$

Where:

$m_i$  = i<sup>th</sup> layer drainage coefficient.

$a_i$  = i<sup>th</sup> layer coefficient.

$d_i$  = i<sup>th</sup> layer thickness (cm).

So long as ASTM D1423 test cannot be carried out to find the modulus of elasticity, the layer coefficient of the asphalt concrete is assumed to be 0.42 (TASB, 2002).

Equation (3) and (4) can be used for determination of the coefficient of the black base or ganular base layers and for ganular subbase layer, respectively.

$$a_{Base} = 0.3356 \log E_{Base} - 0.977 \quad (3)$$

$$a_{Subbase} = 0.3141 \log E_{Subbase} - 0.839 \quad (4)$$

Where  $E_{Base}$  is base elastic modulus and  $E_{Subbase}$  is subbase elastic modulus in kg/cm<sup>3</sup>. If modulus of elasticity is not known, it is possible to use graphs that relate CBR to layer coefficient (TASB, 2002).

Not only should equation (2) be satisfied, but also the thickness of each layer should be such a way that the total compressive stress, applied on lower layers, may be reduced to the tolerable stress of these layers. For this to happen, the following equations should be satisfied.

$$a_1 d_1 \geq 2.54 SN_1 \quad (5)$$

$$a_1 d_1 + m_2 a_2 d_2 \geq 2.54 SN_2 \tag{6}$$

$$a_1 d_1 + m_2 a_2 d_2 + m_3 a_3 d_3 \geq 2.54 SN_3 \tag{7}$$

In these equations,  $SN_1$ ,  $SN_2$  and  $SN_3$  are the structural number of ganular base, ganular subbase and subgrade layers respectively. Their values are found from equation (1) with the only difference that instead of subgrade modulus of elasticity, ganular base modulus of elasticity is used to find  $SN_1$  and ganular subbase modulus of elasticity is used to find  $SN_2$ . Also, layer thickness of asphalt concrete including wearing course, binder course and ganular base should not be taken less than those given in Table (2), considering the construction thickness. For the ganular subbase layer, the minimum thickness is taken as 15 cm.

**Table1:** Standard normal deviates for various levels of reliability(TASB, 2002).

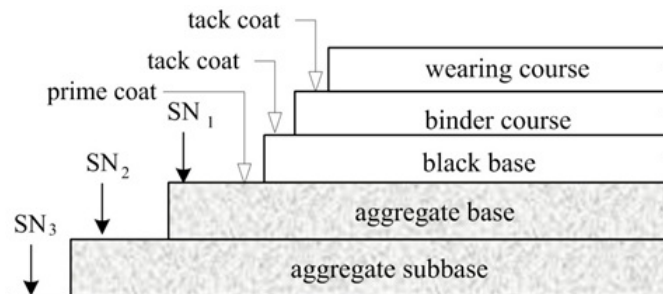
Reliability (%)	Standard normal deviate( $Z_r$ )	Reliability(%)	Standard normal deviate ( $Z_r$ )
50	0.000	93	-1.476
60	-0.253	94	-1.555
70	-0.522	95	-1.645
75	-0.672	96	-1.751
80	-0.841	97	-1.881
85	-1.037	98	-2.054
90	-1.282	99	-2.327
91	-1.340	99.90	-3.090
92	-1.405	99.99	-3.750

**Table2:** Minimum thickness for Asphalt Concrete and Base(TASB, 2002).

Traffic(ESAL)	Asphalt Concrete (cm)	Base (cm)
Less than 150000	Surface Asphalt	10
150000-500000	8	10
500000-1000000	10	10
greater than	12	10

Now we can write the optimization model of the problem to find a unique answer for the thickness of each layer which, in the meantime, is the optimum answer too.

Considering a 5-layer system as shown in Figure (1), the mixed-integer programming model of the problem can be written as follows:



**Fig. 1:** assumed structure of pavement for optimization.

**Objective Function:**

$$\text{Min } Z = LW[C_T d_T + C_B d_B + C_{BB} d_{BB} + \frac{C_{BA} d_{BA}}{100} + \frac{C_{SB} d_{SB}}{100} + E_{BB} C_{TC} d_{TC}] \tag{8}$$

**Subjected to:**

**Van Tile constraints:**

$$a_T d_T + a_B d_B + E_{BT} a_{BT} d_{BT} \geq 2.54 SN_1 E_{BA} \tag{9}$$

$$a_T d_T + a_B d_B + E_{BT} a_{BT} d_{BT} \geq 2.54 SN_3 (1 - E_{BA}) \tag{10}$$

$$a_T d_T + a_B d_B + E_{BT} a_{BT} d_{BT} + m_{BA} a_{BA} d_{BA} \geq 2.54 SN_2 E_{SB} \tag{11}$$

$$a_T d_T + a_B d_B + E_{BT} a_{BT} d_{BT} + m_{BA} a_{BA} d_{BA} \geq 2.54SN_2(1 - E_{SB}) \quad (12)$$

$$a_T d_T + a_B d_B + E_{BT} a_{BT} d_{BT} + m_{BA} a_{BA} d_{BA} + m_{SB} a_{SB} d_{SB} \geq 2.54SN_2(1 - E_{SB}) \quad (13)$$

Constraint for minimum thickness of wearing course:

$$d_T \geq 0.35(d_T + d_B) \quad (14)$$

Constraints for minimum thickness of asphalt concrete, ganular base and ganular subbase:

$$d_T + d_B \geq \min_{T,AC} \quad (15)$$

$$d_{BA} \geq E_{BA} \min_{T,BA} \quad (16)$$

$$d_{SB} \geq E_{SB} \min_{T,SB} \quad (17)$$

Constraint for using of ganular subbase layer in the pavement structure:

$$E_{BB} + E_{BA} \geq E_{SB} \quad (18)$$

Constraints for construction thickness of wearing course, binder course and black base layer:

$$d_T \geq (N_T K_T + \frac{K_T}{4}) \quad (19)$$

$$d_B \geq (N_B K_B + \frac{K_B}{4}) \quad (20)$$

$$d_{BB} \geq E_{BB} (N_{BB} K_{BB} + \frac{K_{BB}}{4}) \quad (21)$$

$$d_T \leq (N_T K_T + \frac{K_T}{2}) \quad (22)$$

$$d_B \leq (N_B K_B + \frac{K_B}{2}) \quad (23)$$

$$d_{BB} \leq E_{BB} (N_{BB} K_{BB} + \frac{K_{BB}}{2}) \quad (24)$$

**Decision Variables:**

$$d_T, d_B, d_{BB}, d_{BA}, d_{SB} \geq 0$$

$$E_{BB}, E_{BA}, E_{SB} \geq \text{binary}$$

$$N_T, N_B, N_{BB} = \text{integer}$$

Where:

Z = cost of pavement construction

L = pavement length (m)

W = pavement width (m)

$d_T$  = thickness of wearing course (cm)

$d_B$  = thickness of binder course (cm)

$d_{BB}$  = thickness of black base layer (cm)

$d_{BA}$  = thickness of ganular base layer (cm)

$d_{SB}$  = thickness of ganular subbase layer (cm)

$d_{TC}$  = thickness of tack coat between black base and binder course (kg/m<sup>2</sup>)

$C_T$  = material and construction cost of 1m<sup>2</sup> of wearing course having a thickness of 1cm

- $C_B$  = material and construction cost of  $1m^2$  of binder course having a thickness of 1cm
- $C_{BB}$  = material and construction cost of  $1m^2$  of black base layer having a thickness of 1cm
- $C_{BA}$  = material and construction cost of  $1m^3$  of ganular base layer
- $C_{SB}$  = material and construction cost of  $1m^3$  of ganular subbase layer
- $C_{TC}$  = cost of  $1m^2$  of tack coating
- $K_T$  = nominal maximum aggregate size (cm) in the wearing course. This size, depending on the case, can be 1.25 or 1.9 cm
- $K_B$  = nominal maximum aggregate size (cm) in the binder course layer. This size, depending on the case, can be 1.9 or 2.5 cm
- $K_{BB}$  = nominal maximum aggregate size (cm) in the black base layer. This size, depending on the case, can be 2.5 or 3.75 cm
- $E_{BB}$  = variable indicating the presence or absence of the black base layer in the optimum structure of the pavement.  $E_{BB} = 1$  means the presence and  $E_{BB} = 0$  means the absence of the black base layer
- $E_{BA}$  = variable indicating the presence or absence of the ganular base layer in the optimum structure of the pavement.  $E_{BA} = 1$  means the presence and  $E_{BA} = 0$  means the absence of the ganular base
- $E_{SB}$  = variable indicating the presence or absence of the ganular subbase layer in the optimum structure of the pavement.  $E_{SB} = 1$  means the presence and  $E_{SB} = 0$  means the absence of the ganular subbase

Constraints (9) to (13) show Van Til conditions in the AASHTO pavement design method. Since binder course is constructed before wearing course and the latter is constructed after smoothing the irregularities of the pavement surface; and, because aggregate sizes in the binder course are greater than those in the wearing course, the binder course thickness is usually taken to be more than that of the wearing course. Also, having in mind that in Iran the cost of wearing course is more than that of the binder course, constraint (14) should be enforced so that the total thickness of asphalt layer may not be considered as the binder course. Constraint (14) shows that only some percentage of the total thickness of the asphalt concrete should be constructed as the binder course. It has been assumed, here in this paper, that the wearing course thickness is, at least, 35 percent of that of the total thickness of asphalt concrete (binder course and wearing course together). This 35 percent may vary with engineering judgment.

Constraints (15) to (17) show the minimum construction thickness constraints for asphalt concrete, ganular base and ganular subbase layers respectively. Constraint (18) states that a pavement cannot have a subbase layer without first having a base layer. In other words, if there is no base layer in pavement structure, it is not possible to use a subbase layer in the pavement structure. Constraints (19) to (24) are to determine different layer thicknesses of wearing course, binder course and black base, depending on the nominal maximum aggregate size used in each layer.

According to Figure (1), if a black base layer is used in the pavement structure, application of an additional tack coating becomes necessary; presence of  $B_{BT} \cdot C_{TC} \cdot D_{TC}$  term in the objective function shows this fact. Also, in this model, if the determination of the optimum pavement structure and optimum layer thickness is the main objective, it is possible to neglect LW coefficient in the objective function, because it does not affect the optimum solution of the model.

A careful study of the model shows that the presence of  $B_{BT} \cdot d_{BT}$  term in the objective function, along with some constraints obtained from the multiplication of two unknown decision variables, causes the model to be a nonlinear one. knowing that  $B_{BT}$  can only be zero or one, to convert this nonlinear model to a linear one, we may first take  $B_{BT} = 0$  (the pavement lacks a black base layer) and write the objective function and other constraints and solve the model and, then again, assume  $B_{BT} = 1$  (the pavement contains a black base layer) and, based on this assumption, write the objective function and other constraints and solve the model again. Then it is possible to find the optimum structure and thicknesses of the layers by a comparison between the construction cost of the optimum pavement of the two cases.

Figure (2) shows the procedure for the solution of the proposed optimization model and determination of the final optimum result found by this method.

## RESULTS AND DISCUSSION

To study the results obtained, and to find the optimum structure and pavement thickness (using Iran's current yearly price-list of materials and construction), the proposed model was solved, assuming design parameters shown in Table (3) and prices shown in Table (4) and considering different values for the strength of subgrade and different design traffic volumes. In this study, the modulus of elasticity of the subgrade has been taken equal to 200 to 1000 kg/cm<sup>2</sup> with increments of 50 kg/cm<sup>2</sup>, showing a wide range of the strength of the subgrade soil, from a very weak to a rather high strength soil. Also, design traffic volume has been assumed to be the number of 80-KN single-axle load applications equal to  $5 \times 10^3$ ,  $10^4$ ,  $10^5$ ,  $10^6$ ,  $3 \times 10^6$  and  $10^7$  which show light, medium and heavy traffic.

**Table 3:** Design parameters for validating of model.

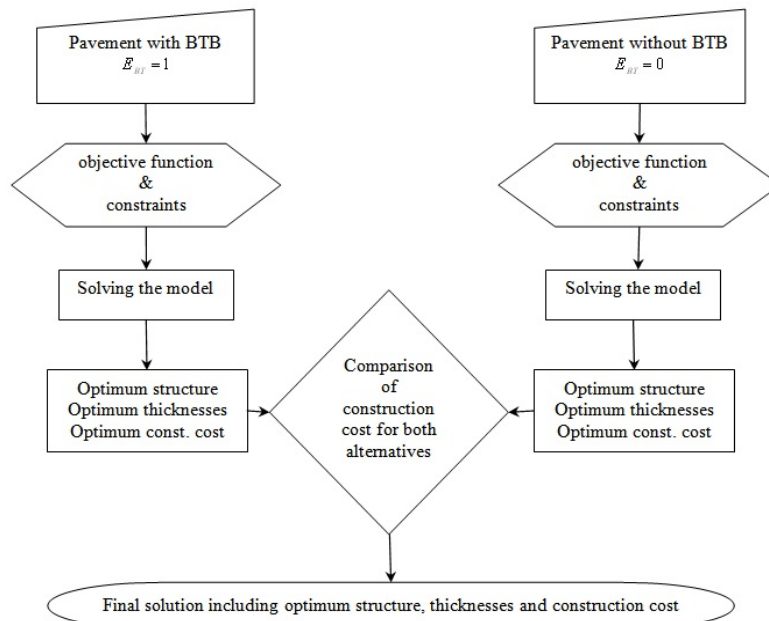
Parameter	Value	Parameter	Value
$a_i$	0.42	$P_i$	4.2
$a_B$	0.42	$P_i$	2.5
$a_{BB}$	0.35	$MI$	-1.645
$E_{Base}$	1925kg/cm <sup>2</sup>	$S_0$	0.35
$E_{Sub\ Base}$	980 kg/cm <sup>2</sup>	$D_{TC}$	0.5Kg
$K_S$	1.25cm	min $T_{Base}$	10cm
$K_B$	1.90cm	min $T_{Sub\ Base}$	10cm
$K_{BT}$	3.75cm		

Drainage coefficient for all layers is assumed to be equal to 1.

**Table 4:** Construction cost for different materials (IMPO, 2007).

Layer	Unit	Unit Cost (Rials)
Wearing course	1m <sup>2</sup> per cm thickness	2500
Binder course	1m <sup>2</sup> per cm thickness	2300
Black Base	1m <sup>2</sup> per cm thickness	1800
ganular Base	m <sup>3</sup>	59280
ganular Subbase	m <sup>3</sup>	18680
Tack Coat	Kg	1950

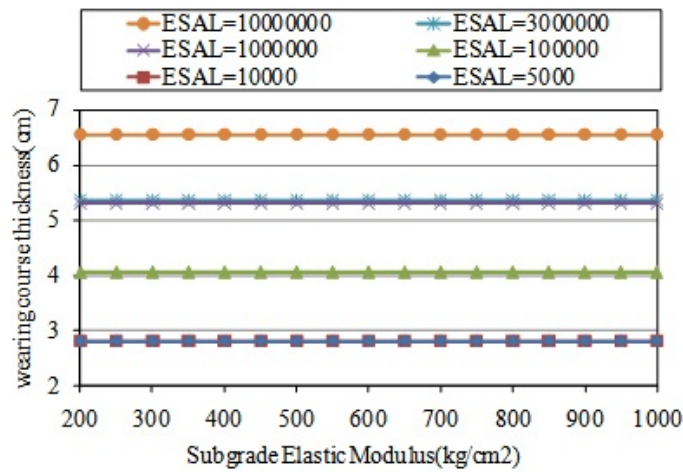
1\$ = 10000Rials



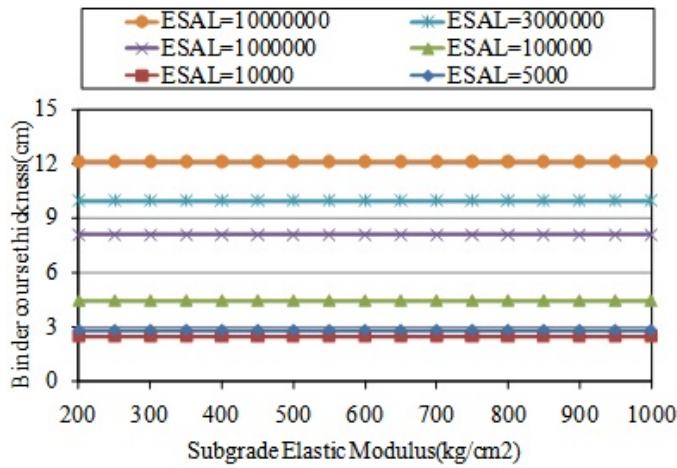
**Fig. 2:** Procedure for solving model to determine the optimum solution.

The results of the model solution and determination of optimum thickness of each layer have been given in Figures (3) to (6). In all the cases, the thickness of the black base layer is zero which means the fact that, with the current Iranian price-list of materials and construction, the use of a black base in the structure of the pavement is not economical. Neither is the use of ganular base and ganular subbase when there is an increase in the strength of the subgrade soil of the pavement.

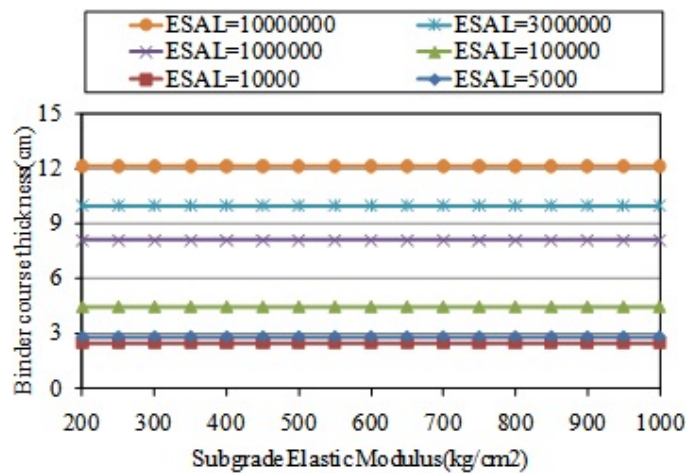
In all the cases, the thickness of the black base layer is zero which means the fact that under present conditions the use of a black base is not economical. Also, with an increase in the strength of the Subgrade soil of the pavement, the use of ganular base and ganular subbase in the structure of the pavement is not economical either. A decrease in traffic volume causes the necessity of the construction of ganular base and ganular subbase layers in lower strengths of the subgrade soil. For example, with traffic of  $5 \times 10^3$ , if the modulus of elasticity of the subgrade soil increases to more than 350 kg/cm<sup>2</sup>, it does not need to use an ganular subbase layer, whereas in a traffic of  $10^7$ , so long as the modulus of elasticity of the subgrade soil has not reached a value of more than 800 kg/cm<sup>2</sup>, the use of an ganular subbase layer in the structure of the pavement is economical. The reason for a sudden fall in the curves related to ganular base and subbase is the presence of constraints (16) and (17) which practically cause the thicknesses of these two layers to be unable to become less than the minimum construction thickness.



**Fig. 3:** Optimum thickness of wearing course.

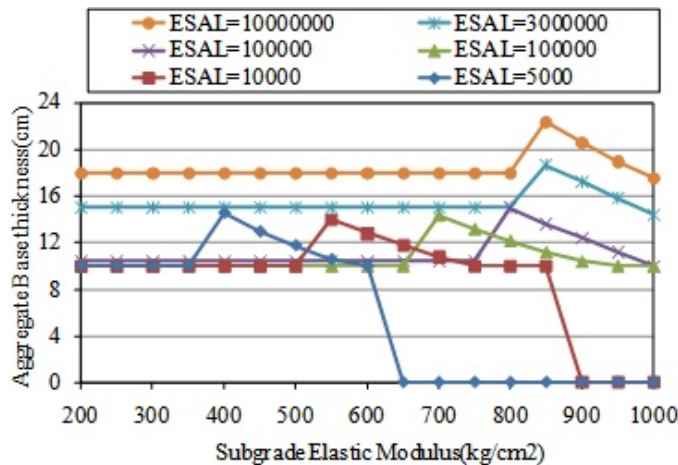


**Fig. 4:** Optimum thickness of binder course.



**Fig. 5:** Optimum thickness of base layer.

Knowing the number of 80-KN equivalent-axle load applications and the modulus of elasticity of the subgrade soil, it is possible to use Figures (3) to (6) for the determination of the optimum thickness of pavement layers. The use of these design charts helps user to find a unique answer for the thickness of each layer- hence the minimum pavement construction cost.



**Fig. 6:** Optimum thickness of subbase layer.

**Conclusion:**

In the present paper, a mixed-integer programming model has been presented for the determination of optimum thickness for each layer of a flexible pavement, based on the method proposed in “Iran highway asphaltic pavements code”, taken from “1993 AASHTO method for the design of flexible pavements”. The use of this model makes possible the determination of the optimum structure and construction thickness of different pavement layers including wearing course, black base, ganular base and ganular subbase, and gives a unique thickness for each layer which, in turn, results in the minimum cost for the construction. This model showed, considering the present cost of materials, the construction of black base layer is not economical. Also, with an increase in the strength of the pavement subgrade, there is no need for ganular and subbase layers (their construction is uneconomical). Moreover, with a decrease in the traffic volume, for lower strengths of the subgrade, there is no more any need for the construction of ganular base and subbase layers either, and the pavement structure is converted to aull-depth asphalt pavement. Using this model, and considering the yearly price-lists, it is possible to draw design graphs to find optimum structure and thickness of pavement layers, with regard to different design parameters, and to use the charts to find the optimum thickness of pavement layers.

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