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A Multistage Adomian Decomposition Method for Solving The Autonomous Van Der Pol System

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Abstract: In this paper, A numeric- analytic method called Multistage Adomian Decomposition Method (MADM) based on an adaptation of classical (ADM) method is applied to the autonomous Van der Pol system . The ADM yields an analytical solution in terms of a rapidly convergent infinite power series with easily computable terms. The ADM is treated as an algorithm for approximating the solution of the problem in a sequence of time Adomian Decomposition method (ADM), intervals (i.e. time steps). Numerical comparisons with the classical ADM, and the classical Rungge – Kutta order four (RK45) methods are presented.

Key word: Runge Van der pol system, Runge – Kutta method (RK45)

INTRODUCTION

The study of nonlinear oscillators has been investigated in the development of the theory of dynamical systems. The Van der Pol oscillator (VPO), described by a second – order nonlinear differential equation which can be regarded as a description of a mass - spring - damper system with a nonlinear position - dependent on damping coefficient or, equivalently, an RLC electrical circuit with a negative - nonlinear resistor, such as electronics, biology or acoustics. It represents a nonlinear system with an interesting behavior that arises naturally in several applications.

This kind of nonlinear oscillator was used by Van der Pol in the 1920s to study oscillations in vacuum tube circuits.

In standard form, it is given by a second – order nonlinear differential equation of type :

$$\ddot{x} + \mu(x^2 - 1)\dot{x} + x = 0, \tag{1}$$

which can be reduced to two dimensional system of first order differential equations

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -x - \mu(x^2 - 1)y \end{aligned} \tag{2}$$

where $\mu\ (\mu > 0)$ is a control parameter that reflects the degree of nonlinearity of the system . In studying the case $(\mu >> 1)$,

Van der Pol discovered the importance of what has been become known as relaxation oscillations (Van der Pol, 1926).

In this paper, we are interested in the analytic Adomian Decomposition Method (ADM) (Adomian, 1988; 1994) to solve the autonomous Van der Pol system.

Unfortunately, the ADM and Power Series Method (PSM) are not guaranteed to give analytic solutions valid globally in time as proved by Répaci (Repaci, 1990). This lack of global convergence can, however, be overcomed by recursively applying ADM over successive time intervals, as first hinted in (Adomian, 1988). This numeric- analytic procedure of ADM, we call (MADM), which has been applied to many important equations, such as the multispecies Lotka–Volterra equations (Olek, 1994; Ruan, 2007), the extended Lorenz

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system (Vadasz and Olek, 2000), the prey– predator problem (Chowdhury *et al.,* 2009) the chaotic Lorenz system (Abdulaziz *et al.,* 2008; Ghosh *et al.,* 2007; Guellal *et al.,* 1997; Hashim *et al.,* 1997), systems of ODEs (Shawagfeh and Kaya, 2004), the classical Chen system (Noorani *et al.,* 2007; Abdulaziz *et al.,* 2008) and the Haldane equation for substrate inhibition enzyme kinetics (Sonnad and Goudar, 2004).

Solution system (2) using (ADM):

To solve the system of equations (2), ADM is employed

$$x(t) = x(t=0) + \int_0^t y\,dt,$$
$$y(t) = y(t=0) + \int_0^t (-x - \mu(x^2-1)y)\,dt, \tag{3}$$

As usual in ADM method the solutions of the system of Eqs. (3) are considered to be as the sum of the following infinite series :

$$x = \sum_{n=0}^{\infty} x_n \quad , \qquad y = \sum_{n=0}^{\infty} y_n, \tag{4}$$

The integral in Eq. (3), as the sum of the following infinite series.

$$f(x,y) = \sum_{n=0}^{\infty} A_n(x_0, x_1, \dots, x_n, y_0, y_1, \dots, y_n), \tag{5}$$

$$g(x,y) = \sum_{n=0}^{\infty} B_n(x_0, x_1, \dots, x_n, y_0, y_1, \dots, y_n), \tag{6}$$

where A_n, B_n are called Adomian Polynomials (Adomian, 1988; 1994). Substituting (4)–(6) into (3) yields :

$$\sum_{n=0}^{\infty} x_n = x(t=0) = \int_0^t \sum_{n=0}^{\infty} A_n(x_0, x_1, \dots, x_n, y_0, y_1, \dots, y_n)\,dt$$
$$= \sum_{n=0}^{\infty} \int_0^t A_n(x_0, x_1, \dots, x_n, y_0, y_1, \dots, y_n)\,dt, \tag{7}$$

and

$$\sum_{n=0}^{\infty} y_n = y(t=0) = \int_0^t \sum_{n=0}^{\infty} B_n(x_0, x_1, \dots, x_n, y_0, y_1, \dots, y_n)\,dt,$$
$$= \sum_{n=0}^{\infty} \int_0^t B_n(x_0, x_1, \dots, x_n, y_0, y_1, \dots, y_n)\,dt, \tag{8}$$

From which we can define the following scheme:

$$x_0 = x(t=0),$$
$$y_0 = y(t=0),$$
$$x_{n+1} = \int_0^t A_n(x_0, x_1, \dots, x_n, y_0, y_1, \dots, y_n)\,dt, \tag{9}$$
$$y_{n+1} = \int_0^t B_n(x_0, x_1, \dots, x_n, y_0, y_1, \dots, y_n)\,dt, \qquad n = 0, 1, 2 \dots\dots$$

Algorithm (Computing Adomian Polynomials):

Input: the system

$G_1 = G(u_1, u_2, \ldots, u_n)$
$G_2 = G(u_1, u_2, \ldots, u_n)$
$\vdots \qquad \vdots$
$G_k = G(u_1, u_2, \ldots, u_n)$
set $n = N$, $m = M$, $k = K$; the input of Adomian Polynomials needed .

Output: A_{ji}; the Adomian Polynomials

Step 1: set $i=1$

Step 2: while $i \leq n$ we have:

$$u_i(\lambda) = \sum_{b=0}^{m} u_{ib}\lambda^b$$

Step 3: set $j=1$
Step 4: while $j \leq n$ do steps (5) and (6)

Step 5: $G_j(\lambda) = G_j(u_j(t) = u_j(\lambda))$

Step 6: $G_i = G_j(\lambda)$

Step 7: $s_i =$ expansion of $G_j(\lambda)$ w.r.t. λ

$ft_i = s_i(\lambda)$

Step 8: while $j \leq k$ and while $j \leq m$

$$A_{ji} = \frac{\partial^i}{\partial \lambda^i}(ft_j)(0)/(i)! = D^i(ft_j)(0)/(i)!$$

Step 9: output Aji (the Adomian Polynomials)
Step 10: end.

Computing Adomian Polynomials for system (2):

Computing Adomian Polynomial by Algorithm (2.1) yields to:

$$\begin{aligned} &A_0(x_0, y_0) = y_0 \\ &A_1(x_0, x_1, y_0, y_1) = y_1 \\ &A_2(x_0, x_1, x_2, y_0, y_1, y_2) = y_2 \\ &A_3(x_0, x_1, x_2, x_3, y_0, y_1, y_2, y_3) = y_3 \\ &\vdots \end{aligned} \tag{10.a}$$

and

$$\begin{aligned} B_0(x_0, y_0) &= -x_0 - \mu(x_0^2 - 1)y_0 \\ B_1(x_0, x_1, y_0, y_1) &= -x_1 - 2D(\mu)(x_0^2 - 1)x_0x_1y_0 - \mu(x_0^2 - 1)y_1 \\ B_2(x_0, x_1, x_2, y_0, y_1, y_2) &= -x_2 - D(\mu)(x_0^2 - 1)x_1^2y_0 - 2D^{(2)}(\mu)(x_0^2 - 1)x_0^2x_1^2y_0 \\ &\quad - 2D(\mu)(x_0^2 - 1)x_0x_1y_1 - \mu(x_0^2 - 1)y_2 \\ &\quad - 2D(\mu)(x_0^2 - 1)x_0x_2y_0 \end{aligned}$$

$$\begin{aligned}B_3(x_0, x_1, x_2, x_3, y_0, y_1, y_2, y_3) = &-x_3 - D(\mu)(x_0^2-1)x_1^2 y_1 - 2D^{(2)}(\mu)(x_0^2-1)x_0^2 x_1^2 y_1\\ &-2D(\mu)(x_0^2-1)x_0 x_1^3 y_0 - 2D(\mu)(x_0^2-1)x_0 x_1 y_2\\ &-\frac{4}{3}D^{(3)}(\mu)(x_0^2-1)x_0^3 x_1^3 y_0 - \mu(x_0^2-1)y_3\\ &-2D(\mu)(x_0^2-1)x_0 x_3 y_0 - 2D(\mu)(x_0^2-1)x_0 x_2 y_1\\ &-2D(\mu)(x_0^2-1)x_1 x_2 y_0 - 4D^{(2)}(\mu)(x_0^2-1)x_0^2 x_1 x_2 y_0\end{aligned} \tag{10.b}$$

$\vdots$

now substituting (10) in (9) yields :

$$x_1 = \int_0^t A_0(x_0, y_0)dt = y_0 t,$$

$$y_1 = \int_0^t B_0(x_0, y_0)dt = (-x_0 - \mu(x_0^2-1)y_0)t,$$

$$x_2 = \int_0^t A_1(x_0, x_1, y_0, y_1)dt = \frac{1}{2}(-x_0 - \mu(x_0^2-1)y_0)t^2,$$

$$\begin{aligned}y_2 = \int_0^t B_1(x_0, x_1, y_0, y_1)dt = &\frac{1}{2}(-y_0 - 2D(\mu)(x_0^2-1)x_0 y_0^2)\\ &-\mu(x_0^2-1)(-x_0 - \mu(x_0^2-1)y_0))t^2,\end{aligned}$$

$$\begin{aligned}x_3 = \int_0^t A_2(x_0, x_1, x_2, y_0, y_1, y_2)dt = &\frac{1}{6}(-y_0 - 2D(\mu)(x_0^2-1)x_0 y_0^2)\\ &-\mu(x_0^2-1)(-x_0 - \mu(x_0^2-1)y_0))t^3,\end{aligned}$$

$$\begin{aligned}y_3 = \int_0^t B_2(x_0, x_1, x_2, y_0, y_1, y_2)dt = &\frac{1}{3}(\frac{-1}{2}(-x_0 - \mu(x_0^2-1)y_0)\\ &-D(\mu)(x_0^2-1)y_0^3 - 2D^{(2)}(\mu)(x_0^2-1)x_0^2 y_0^3\\ &-2D(\mu)(x_0^2-1)x_0 y_0(-x_0 - \mu(x_0^2-1)y_0)\\ &-\frac{1}{2}\mu(x_0^2-1)(-y_0 - 2D(\mu)(x_0^2-1)x_0 y_0^2\\ &-\mu(x_0^2-1)(-x_0 - \mu(x_0^2-1)y_0)))t^3\end{aligned}$$

$\vdots$

and so on …

The four terms of the approximations to the solutions are considered as :

$x(t) \approx x_0 + x_1 + x_2 + x_3$

$y(t) \approx y_0 + y_1 + y_2 + y_3$.

for the convergence of the method, we refer the reader to (Babolian and Biazar, 2000) in which the problem of convergence has been discussed briefly .

Solution of the system (2) by MADM:

We consider the general system:

$$x_i' = \sum_{j=1}^{2} a_{ij} x_j + \sum_{p=1}^{2}\sum_{q=1}^{2} a_{ipq} x_p^2 x_q \qquad , \ i = 1,2 \tag{11}$$

where $x_1' = x'(t) = \dot{x}$ and $x_2' = y'(t) = \dot{y}$ the prime denotes differentiation with respect to time.

If we denote the linear term (the first term on the R.H.S.) as R_1 and the nonlinear term (the second term) as R_{i2} , then we can write the above system of equations in an operator form:

$$Lx_i = R_1 + R_{i2} \qquad , \quad i = 1,2 \tag{12}$$

where *L* is the differential operator $d(.)/dt$. Applying the inverse (integral) operator L^{-1} to (12) we obtain

$$x_i(t) = x_i(t=0) + L^{-1}R_{i1} + L^{-1}R_{i2} \qquad , \quad i = 1,2 \tag{13}$$

According to the ADM (Adomian, 1988; 1994), the solutionwill $x_i(t)$ be given by the series

$$x_i(t) = \sum_{n=0}^{\infty} x_{in}(t) \qquad , \ i = 1,2 \tag{14}$$

Bearing this in mind, the linear term R_{i1} then becomes

$$R_{i1} = \sum_{j=1}^{2}\sum_{n=0}^{\infty} a_{ij} x_{in}, \tag{15}$$

so that $L^{-1}R_{i1}$ will be given by

$$L^{-1}R_{i1} = \sum_{j=1}^{2}\sum_{n=0}^{\infty} a_{ij} \int_0^t x_{in} dt \qquad , \ i = 1,2 \tag{16}$$

The non-linear term R_{i2} is decomposed as follows :

$$R_{i2} = \sum_{p=1}^{2}\sum_{q=1}^{2} a_{ipq} \sum_{n=0}^{\infty} A_{in,p,q}, \tag{17}$$

where the $A_{in,p,q}$'s are the so-called Adomian Polynomials, computed by using Algorithm (2.1). In this case Adomian Polynomial will be given by the formula

$$A_{in,p,q} = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[M\left(\sum_{k=0}^{\infty} \lambda^k x_{kp}^2 \sum_{k=0}^{\infty} \lambda^{k)} x_{kq}\right)\right]_{\lambda=0} \tag{18}$$

where $M(x,y)=x^2y$ for each n=0,1,2,... Moreover, $L^{-1}R_{i2}$ will be given by

$$L^{-1}R_{i2} = \sum_{p=1}^{2}\sum_{q=1}^{2} a_{iipq} \sum_{n=0}^{\infty} \int_0^t A_{in,p,q} dt, \tag{19}$$

Putting (14),(16),(17) into (13) yields for each *i*=1,2

$$\sum_{n=0}^{\infty} x_{in}(t) = x_i(t=0) + \sum_{j=1}^{2}\sum_{n=0}^{\infty} a_{ij} \int_0^t x_{in} dt + \sum_{p=1}^{2}\sum_{q=1}^{2} a_{ipq} \sum_{n=0}^{\infty} \int_0^t A_{in,p,q} dt, \tag{20}$$

Consequently, for each *i*=1,2 we have

$$x_{i0} = x_i(t=0), \tag{21}$$

$$x_{i1} = \sum_{j=1}^{2} a_{ij} \int_0^t x_{i0}\,dt + \sum_{p=1}^{2}\sum_{q=1}^{2} a_{ipq} \sum_{n=0}^{\infty} \int_0^t A_{i0,p,q}\,dt, \tag{22}$$

$$x_{i2} = \sum_{j=1}^{2} a_{ij} \int_0^t x_{i1}\,dt + \sum_{p=1}^{2}\sum_{q=1}^{2} a_{ipq} \sum_{n=0}^{\infty} \int_0^t A_{i1,p,q}\,dt, \tag{23}$$

$$\vdots \qquad\qquad \vdots$$

$$x_{i,n+1} = \sum_{j=1}^{2} a_{ij} \int_0^t x_{in}\,dt + \sum_{p=1}^{2}\sum_{q=1}^{2} a_{ipq} \sum_{n=0}^{\infty} \int_0^t A_{in,p,q}\,dt. \tag{24}$$

Now by using only the nonlinear terms of the input functions in Algorithm (2.1) from equation (18) we compute the Adomian Polynomials as follows:

$$\begin{aligned} A_{10,p,q} &= 0\\ A_{11,p,q} &= 0\\ A_{12,p,q} &= 0\\ A_{13,p,q} &= 0\\ A_{14,p,q} &= 0\\ &\vdots \end{aligned} \tag{25.a}$$

and

$$\begin{aligned} A_{23,p,q} = &-\frac{1}{6} D^{(3)}(\mu)(x_{10}^2 x_{20})(2x_{10}x_{20}x_{11} + x_{10}^2 x_{21})^3\\ &-\frac{1}{2} D^{(2)}(\mu)(x_{10}^2 x_{20})(2x_{10}x_{20}x_{11} + x_{10}^2 x_{21})(2x_{11}^2 x_{20} + 4x_{10}x_{21}x_{11} + 4x_{10}x_{20}x_{12}\\ &+ 2x_{10}^2 x_{22}) - -\frac{1}{6} D(\mu)(x_{10}^2 x_{20})(12x_{11}x_{20}x_{12} + 6x_{11}^2 x_{21} + 12x_{10}x_{22}x_{11} +\\ &+ 12x_{10}x_{21}x_{12} + 12x_{10}x_{20}x_{13} + 6x_{10}^2 x_{23}) \end{aligned}$$

$$\begin{aligned} A_{24,p,q} = &-\frac{1}{24} D^{(4)}(\mu)(x_{10}^2 x_{20})(2x_{10}x_{20}x_{11} + x_{10}^2 x_{21})^4\\ &-\frac{1}{4} D^{(3)}(\mu)(x_{10}^2 x_{20})(2x_{10}x_{20}x_{11} + x_{10}^2 x_{21})^2 (2x_{11}^2 x_{20} + 4x_{10}x_{21}x_{11}\\ &+ 4x_{10}x_{20}x_{12} + 2x_{10}^2 x_{22}) - \frac{1}{8} D^{(2)}(\mu)(x_{10}^2 x_{20})(2x_{11}^2 x_{20} + 4x_{10}x_{21}x_{11}\\ &+ 4x_{10}x_{20}x_{12} + 2x_{10}^2 x_{22}) - \frac{1}{6} D^{(2)}(\mu)(x_{10}^2 x_{20})(2x_{10}x_{20}x_{11} + x_{10}^2 x_{21})(12x_{10}x_{20}x_{12}\\ &+ 6x_{11}^2 x_{21} + 12x_{10}x_{22}x_{11} + 12x_{10}x_{21}x_{12} + 12x_{10}x_{20}x_{13} + 6x_{10}^2 x_{23})\\ &-\frac{1}{24} D(\mu)(x_{10}^2 x_{20})(24x_{12}^2 x_{20} + 48x_{11}x_{21}x_{12} + 48x_{11}x_{20}x_{13} + 24x_{11}^2 x_{22}\\ &+ 48x_{10}x_{23}x_{11} + 48x_{10}x_{22}x_{12} + 48x_{10}x_{21}x_{13} + 48x_{10}x_{20}x_{14} + 24x_{10}^2 x_{24})\\ &\vdots \end{aligned} \tag{25.b}$$

and so on

Since $A_{n,p,q} = 0$, $\forall\ n = 0,1,2,\ldots$

$\therefore \quad \int_0^t A_{1n,p,q} = 0$, $\forall\ n = 0,1,2,\ldots$ and hence

$$x_{10} = x_1(t=0) \tag{26}$$

$$x_{11} = \sum_{j=1}^{2} a_{1j} \int_0^t x_{10}\,dt \tag{27}$$

$$x_{12} = \sum_{j=1}^{2} a_{1j} \int_0^t x_{11}\,dt \tag{28}$$

$$x_{1,n+1} = \sum_{j=1}^{2} a_{1j} \int_0^t x_{1n}\,dt \tag{29}$$

also

$$\int_0^t A_{20,p,q}\,dt = -\mu(x_{10}^2 x_{20})t$$

$$\int_0^t A_{21,p,q}\,dt = \frac{1}{2}(-D(\mu)(x_{10}^2 x_{20})(2x_{10}x_{20}x_{11} + x_{10}^2 x_{21}))t^2$$

$$\int_0^t A_{22,p,q}\,dt = \frac{1}{3}(-\frac{1}{2}D^{(2)}(\mu)(x_{10}^2 x_{20})(2x_{10}x_{20}x_{11} + x_{10}^2 x_{21})^2$$
$$-\frac{1}{2}D(\mu)(x_{10}^2 x_{20})(2x_{11}^2 x_{20} + 4x_{10}x_{21}x_{11} + 4x_{10}x_{20}x_{12} + 2x_{10}^2 x_{22}))t^3$$

$$\int_0^t A_{23,p,q}\,dt = \frac{1}{4}(-\frac{1}{6}D^{(3)}(\mu)(x_{10}^2 x_{20})(2x_{10}x_{20}x_{11} + x_{10}^2 x_{21})^3$$
$$-\frac{1}{2}D^{(2)}(\mu)(x_{10}^2 x_{20})(2x_{10}x_{20}x_{11} + x_{10}^2 x_{21})(2x_{11}^2 x_{20} + 4x_{10}x_{21}x_{11}$$
$$+4x_{10}x_{20}x_{12} + 2x_{10}^2 x_{22}) - \frac{1}{6}D(\mu)(x_{10}^2 x_{20})(12x_{11}x_{20}x_{12} + 6x_{11}^2 x_{21}$$
$$+12x_{10}x_{22}x_{11} + 12x_{10}x_{21}x_{12} + 12x_{10}x_{20}x_{13} + 6x_{10}^2 x_{23}))t^4$$

$\vdots$

hence we obtain

$$x_{20} = x_2(t=0) \tag{30}$$

$$x_{21} = \sum_{j=1}^{2} a_{2j} \int_0^t x_{20} dt + \sum_{p=1}^{2}\sum_{q=1}^{2} a_{2pq} (-\mu(x_{10}^2 x_{20})t) \tag{31}$$

$$x_{22} = \sum_{j=1}^{2} a_{2j} \int_0^t x_{21} dt + \sum_{p=1}^{2}\sum_{q=1}^{2} a_{2pq} (\frac{1}{2}(-D(\mu)(x_{10}^2 x_{20})(2x_{10}x_{20}x_{11} + x_{10}^2 x_{21}))t^2) \tag{32}$$

$$\begin{aligned} x_{23} = \sum_{j=1}^{2} a_{2j} \int_0^t x_{22} dt + \sum_{p=1}^{2}\sum_{q=1}^{2} a_{2pq} (\frac{1}{3}(-\frac{1}{2}D^{(2)}(\mu)(x_{10}^2 x_{20})(2x_{10}x_{20}x_{11} + x_{10}^2 x_{21})^2 \\ -\frac{1}{2}D(\mu)(x_{10}^2 x_{20})(2x_{11}^2 x_{20} + 4x_{10}x_{21}x_{11} + 4x_{10}x_{20}x_{12} + 2x_{10}^2 x_{22}))t^3) \end{aligned} \tag{33}$$

$$\begin{aligned} x_{24} = \sum_{j=1}^{2} a_{2j} \int_0^t x_{23} dt + \sum_{p=1}^{2}\sum_{q=1}^{2} a_{2pq} \frac{1}{4}(-\frac{1}{6}D^{(3)}(\mu)(x_{10}^2 x_{20})(2x_{10}x_{20}x_{11} + x_{10}^2 x_{21})^3 \\ -\frac{1}{2}D^{(2)}(\mu)(x_{10}^2 x_{20})(2x_{10}x_{20}x_{11} + x_{10}^2 x_{21})(2x_{11}^2 x_{20} + 4x_{10}x_{21}x_{11} \\ + 4x_{10}x_{20}x_{12} + 2x_{10}^2 x_{22}) - \frac{1}{6}D(\mu)(x_{10}^2 x_{20})(12x_{11}x_{20}x_{12} + 6x_{11}^2 x_{21} \\ + 12x_{10}x_{22}x_{11} + 12x_{10}x_{21}x_{12} + 12x_{10}x_{20}x_{13} + 6x_{10}^2 x_{23}))t^4) \end{aligned} \tag{34}$$

and so on

Upon calculating the polynomials (18) and integrating , one then has for all :

$$x_i(t) = \sum_{n=0}^{\infty} d_{in} \frac{t^n}{n!} \quad , \quad i = 1,2 \quad , \tag{35}$$

where the coefficients d_{in} are given by

$$d_{i0} = x_i(t), \tag{36}$$

$$d_{in} = \sum_{j=1}^{2} a_{ij} d_{j(n-1)} + (n-1)! \sum_{p=1}^{2}\sum_{q=1}^{2}\sum_{k=0}^{n} a_{ipq} \frac{d_{qk}^2}{k!} \frac{d_{p(n-k-1)}}{k!(n-k-1)!} \quad ; n \geq 1 \tag{37}$$

Hence from (35)-(37), the explicit solution to the autonomous Van der Pol system (2) will be given as

$$x = \sum_{n=0}^{\infty} a_n \frac{t^n}{n!} \quad , \quad n \geq 1 \tag{38}$$

$$y = \sum_{n=0}^{\infty} b_n \frac{t^n}{n!} \quad , \quad n \geq 1 \tag{39}$$

where the coefficients are given by the recurrence relations

$$a_0 = x(t=0) \quad , \quad b_0 = y(t=0), \tag{40}$$

$$\begin{aligned} a_1 &= y_0 \\ b_1 &= -x_0 - \mu(x_0^2 - 1)y_0 \end{aligned} \tag{41}$$

$\vdots$

$$a_n = b_{n-1} \qquad , n \geq 1 \tag{42}$$

$$b_n = -a_{n-1} + \mu b_{n-1} - \mu(n-1)!\sum_{k=0}^{n-1} \frac{a_k^2 b_{n-k-1}}{k!(n-k-1)!} \qquad , n \geq 1$$

Numerical Results:

The numerical algorithm given by (38)-(42) is computed using Matlab. We consider the following three cases :

Case 1: μ=2, and the initial conditions $x(0) = x_0 = 0.1$ and $y(0) = y_0 = 1$

Case 2: μ=5, and the initial conditions $x(0) = x_0 = 0.1$ and $y(0) = y_0 = 1$

Case 3: μ=10, and the initial conditions $x(0) = x_0 = 0.1$ and $y(0) = y_0 = 1$

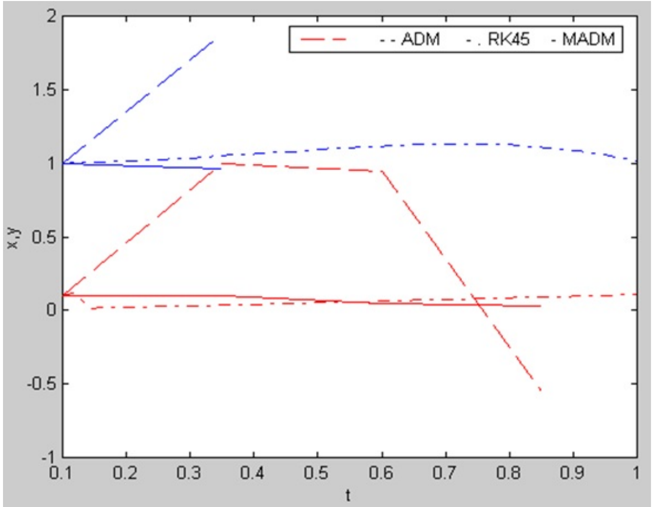

Fig. 4.1: Comparison of the solution of system (2) x and y at time t for case 1 using 4-term classical ADM, 4-term MADM and RK45.

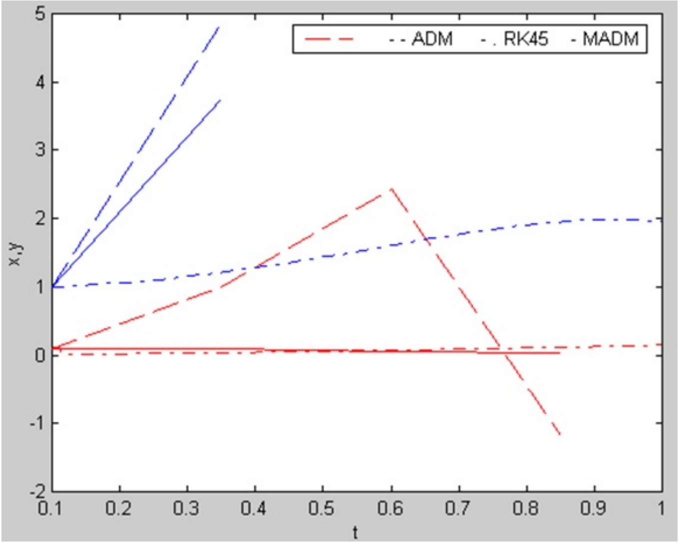

Fig. 4.2: Comparison of the solution of system (2) x and y at time t for case 2 using 4-term classical ADM,4-term MADM and RK45.

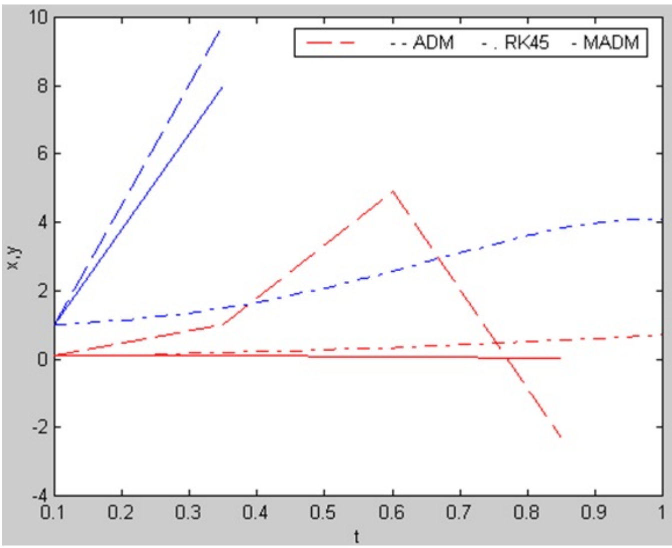

Fig. 4.3: Comparison of the solution of system (2) x and y at time t for case 3 using 4-term classical ADM, 4-term MADM and RK45.

As shown in Fig. (4.1), Fig.(4. 2) and Fig. (4. 3), these 4-term ADM solutions are not accurate enough. However, our 4-term MADM solutions agree very well with the RK45 solutions.

Conclusions:

In this paper, the autonomous Van der Pol system was solved accurately by MADM. The method has the advantage of giving the form of the numerical solution within each time interval which is not possible in purely numerical techniques like RK45. The present technique offers an explicit time-marching algorithm that works accurately over such a bigger time step than the RK45.

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