

Modelling of the Compressive and Tensile Strength Relationship of Concrete in Studies on the Punching Phenomenon in Prestressed Flat Slabs

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Abstract: The present paper addresses the problem of the design of suitable model concrete mixes for the investigation of the punching strength of post-tensioned concrete flat slabs. A number of alternative experimental approaches ranging from the use of lightweight aggregates to a combination of crushed basalt and two grades of sand are considered. It was found that the latter method resulted in concrete strength relationships which were similar to those given in the ACI and CEB-FIP codes. However for such model mixes, information regarding water-cement and aggregate-cement ratios should be obtained from standard prototype design and scaled down grading curves in order to achieve the desired workability. Furthermore, scaled down loading rates are desirable for testing the model cubes and cylinders in order to obtain representative results.

Key words: Concrete, compressive, tensile, strength, model, scale.

INTRODUCTION

The punching phenomenon in flat slabs constitutes a major disadvantage of these structures especially for the flat plate variety employing neither drop panels nor column capitals. Punching failures can initiate progressive collapse and in some cases cause a global structural collapse^[1-2]. Since punching occurs suddenly, often prior to the formation of an overall yield line mechanism, this type of failure is to be avoided.

Several investigators have developed methods for predicting the punching loads of post-tensioned slabs. These methods have been largely semi-empirical, based on tests involving the use of scaled models. In recent years the range and scope of such model tests have been quite diverse and are well documented in literature^[3-11]. However, prototype concrete cannot normally be replaced by any other material in ultimate load tests, and the engineering properties of the hardened cement – aggregate mass that comprises prototype and model concrete are dependent on several factors whose effects and their interaction are not completely understood^[12].

In punching shear studies, it is generally accepted that the compressive strength of concrete and more importantly, the tensile strength, influences the behaviour of a structure in many ways including the strength in diagonal tension and resistance to shear, the cracking load levels and crack patterns and the effective stiffness of the structure and the degree of non-linearity in response to load. Like the compressive

strength, the indirect tensile strength of concrete is dependent on the following variables – method and rate of loading, specimen size, maximum size of aggregate, water-cement ratio, aggregate-cement ratio and the effects of strain gradient amongst others^[13].

The tensile strength of a model concrete is usually higher than that of prototype concrete. It is therefore important to understand the tensile properties of model concrete in the overall modelling process. The aim of the present paper is thus to propose a method of designing suitable model concrete mixes for the study of the punching problem. However, the method adopted is expected to be of much wider application in structural concrete modelling.

MATERIALS AND METHOD

Preliminary Investigations: Initial investigations were conducted using the procedures outlined in^[14], however for the design of the prototype concrete mix the approach in^[15] was adopted. For this mix, crushed basalt aggregate (sizes varying from 19mm to 2.4mm) was combined with a natural Zone 2 sand^[16]. Grading curve No. 2 in^[15] was used to arrive at the proportions given in Table 1.

In the design of the model concrete mix, it was considered essential to comply with the similitude requirements for true models so that the experimental results obtained would be meaningful and a comparison could be made with actual prototype behaviour. As it was realized from the onset that scaling of the fines in

the prototype concrete mix would be difficult, it was necessary to investigate the characteristics of concretes with different types of aggregates using trial mixes for the prototype concrete.

Apart from satisfying the requirement for the prototype concrete of a minimum compressive strength f_{cu} of 35 MPa at 28 days using ordinary Portland cement, several other factors had to be considered for the model concrete mix. Firstly, the maximum aggregate size in the prototype mix had to be directly scaled. Hence, the selection of an appropriate scale for the model bearing in mind overall costs and the need to reduce to a minimum the existence of scale or size effects^[17] was important. Consequently, a $1/3$ scale was chosen and thus the largest size of aggregate for the model mix was 6mm. It was assumed here that the scale ratio between the maximum-size aggregates of model and prototype materials is the same ratio as the mean sizes of the two, which in practice is usually the case.

Secondly, it was considered impractical to model any of the standard consistency apparatus and thus the workability of the fresh model concrete mix was used as a visual measure of its consistency. It was important that the mix was sufficiently workable so that good compaction could be obtained. A smooth surface finish greatly eases the detection of cracks in ultimate load tests.

A major factor in short-term punching tests is to ensure that the relationship between the tensile and compressive strength, f_t and f_{cu} respectively, are identical for both the prototype and model concretes. This relationship has been studied by several investigators^[18-20] Based on their findings and on the types of aggregates locally available, the following equations were proposed:

$$f_t = 0.55 (f_{cu})^{1/2} \quad (1)$$

$$\text{and } f_t = 0.3 (f_{cu})^{2/3} \quad (2)$$

These equations are equivalent expressions and are similar to the ACI and CEB-FIP code estimates^[21-22].

It was stated earlier that a number of alternatives were proposed as aggregates for the model concrete. These are now considered in detail below.

Lightweight Aggregate Concrete: It is generally accepted that lightweight aggregate concrete exhibits tensile strengths which are of the order of 70% to 90%, that of concretes made from crushed rock aggregates^[23-24]. Its use was sought because, in general, the greater concentration of fines in a normal concrete model mix relative to the prototype results in a much higher $f_t/(f_{cu})^{1/2}$ ratio.

Two types of lightweight aggregates were locally available, both of the pulverized-fuel ash variety and manufactured by Lytag Ltd^[25]. These were a 6mm single size aggregate and a fine aggregate with sizes varying from 5mm to 0mm. The aggregates were generally rounded and of bulk density 835 kg/m³ and 1040 kg/m³ respectively. They possessed a higher absorption capacity than normal weight concrete aggregates.

The initial mix design was based on recommendations given in^[26] and design data from Lytag Ltd. Proportioning was carried out on a volume basis, the least total volume of aggregates being required. On account of the high absorption capacity, the aggregates were mixed with about 50% of the mixing water prior to the addition of the cement. After each trial mix, the wet density of the fresh concrete was determined. The results are presented in Table 2, and it is apparent that the concrete strengths are quite consistent and that the tensile strengths f_t are generally less than the value specified for the design (see equation (1)).

In spite of the above however, the use of lightweight aggregate concrete was dispensed with as a final solution because little was known about its behaviour in punching shear. Also for all trial mixes, it was difficult to obtain a good surface finish. Furthermore, the high absorption capacity meant that incorrect batching could result on account of the high moisture content of the aggregates (for illustration compare mixes 3 and 4 in Table 2).

Partial sand replacement of the fines and oven drying of the lightweight aggregates prior to mixing could have alleviated some of the difficulties encountered. However, these measures would have been very laborious and could in turn introduce additional problems.

Crushed Basalt Aggregate Combined with a Bs 882 [16] Zone 2 Sand: In order to obtain a representative model mix, the maximum aggregate size of the prototype concrete had to be directly scaled down. For this purpose 6mm crushed basalt was used, being available locally. The gradings of this aggregate and the fine aggregate, a Zone 2 sand, are given in Table 3.

There is very little information in the literature regarding the water-cement (w/c) and aggregate-cement (a/c) ratios for scaled down concrete mixes. Figure 1 shows a standard design and some scaled down grading curves for medium workability concrete mixes based on the use of angular aggregates. In order to achieve the required design strength, bearing in mind that the water-cement ratio for the prototype mix was 0.46, it was decided to work within a zone in Figure 1

bounded by $3.0 \leq a/c \leq 3.5$ and $0.45 \leq w/c \leq 0.53$. A series of trial mixes involving various combinations of aggregate-cement and water-cement ratios were carried out. Typical combined gradings based on proportions of fines to total aggregate of 31%, 45% and 58% by weight are given in Table 4.

Preliminary tests were based on the data in Table 4. Grading C3 results in a mix with an aggregate-cement ratio of 2.9 compared with 3.4 for Grading C1 to compensate for its higher sand content. However the average 28 day compressive strength based on Grading C3 was 56 MPa which was much higher than the design value of 35 MPa. Grading C1 resulted in a very harsh mix as expected due to the very low proportion of fines passing a 2.4mm sieve size, while Grading C3 produced a very good surface finish.

Crushed Basalt Aggregate Combined with a Twin Grade Sand: From the preceding observations it was obvious that a possible solution to the problem of design of a suitable model mix was in suitably reducing the proportions of very fine sand in order to lower to some extent the 28 day compressive and tensile strengths of the concrete, while at the same time ensuring that the fraction of aggregates passing a 2.4mm sieve was in the range 50% to 55% in order to produce a sufficiently workable mix with smooth surface finish. Hence the required gradings should be similar to Grading C1 in Table 4 for fractions smaller than 600 μ m sieve size, but similar to Grading C3 for fractions of 2.4mm sieve size or larger.

Once it was decided to reduce the proportions of very fine sand, a search was conducted for suitable local alternative. After experimenting with several trial mixes it was found that a combination of a locally available coarse grade sand (or grit) with a small proportion of the BS 882 Zone 2 sand, when used with the 6mm basalt, produced a mix having a good surface finish and medium workability. The grading of the coarse sand or grit is shown in Table 3.

RESULTS AND DISCUSSION

Results: For model concrete made from crushed basalt aggregate combined with a twin grade sand, details of the trial mixes for the investigations are given in Table 5. Altogether a total of twenty-eight mixes were made. From Figure 2 it is apparent that the use of the recommended grading – Grading CS (see also Table 5) leads to a sharp reduction in the fines content compared with Gradings C2 and C3 based on a single grade sand. Furthermore, in contrast with Grading C1, a mix having very good surface finish results.

In order to assess the degree of control obtained with the chosen mix for modelling the compressive and tensile strengths of concrete in punching shear investigations, ten 100mm and fourteen 50mm cubes ($2/3$ and $1/3$ scale respectively), and eight 100mm x 200mm cylinders and ten 50mm x 100mm cylinders ($2/3$ and $1/3$ scale respectively) were tested for each model. The compressive and split cylinder strengths of the model mixes as well as that of the prototype are presented in Table 6.

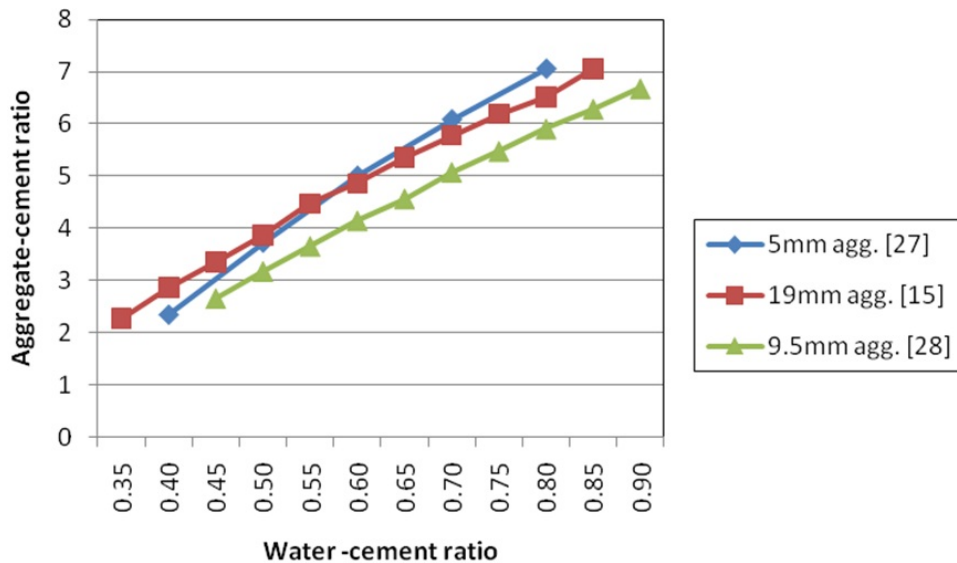


Fig. 1: Curves for medium workability mix based on angular aggregates

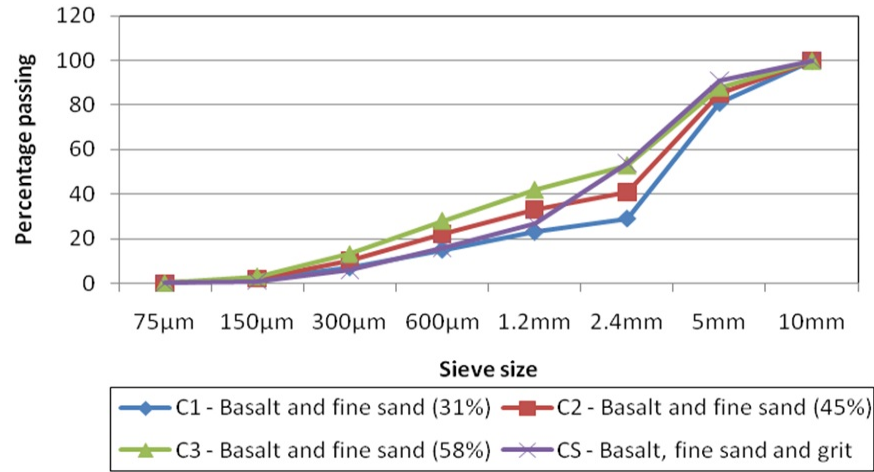


Fig. 2: Combined aggregate grading curves

Table 1: Details of prototype concrete mix*

Water-cement ratio	Aggregate-cement ratio	Ratio of fine to coarse aggregate [†]
0.46	5.1	0.33

* All ratios are by weight

[†] 35% of combined aggregate pass through a 5mm sieve

Table 2: Details of lightweight aggregate trial mixes

Mix	Water-cement ratio	Total aggregate-cement ratio	Fine aggregate-cement ratio	Average cube* compressive strength, f_{cu} (MPa)	Average* split cylinder tensile strength, f_t (MPa)	$\frac{f_t}{\sqrt{f_{cu}}}$
1	0.62	1.9	0.99	39.4	3.51	0.56
2	0.77	2.1	1.0	43.4	3.07	0.47
3	0.73	1.9	0.97	40.0	3.09	0.49
4	0.60	1.9	0.97	36.2	2.92	0.49
5	0.71	2.2	0.80	32.0	3.21	0.57

* Based on 8-12 test specimens of $1/3$ scale

Table 3: Sieve analysis of aggregates

Aggregate	Proportion of aggregates passing the sieve sizes shown (%)							
	10mm	5mm	2.4mm	1.2mm	600µm	300µm	150µm	75µm
A1	100	73	1.6	0.6	0	0	0	0
A2	100	99	90	72	49	23	5	0
A3	100	99	71	23	11	3	0	0

A1 16mm crushed basalt
A2 Zone 2 sand
A3 Coarse grit

Table 4: Combined gradings for crushed basalt and zone 2 sand

Grading	Fraction of aggregates passing the sieve sizes shown (%)							
	10mm	5mm	2.4mm	1.2mm	600µm	300µm	150µm	75µm
C1	100	81	29	23	15	7	2	0
C2	100	85	41	33	22	10	2	0
C3	100	88	53	42	28	13	3	0

C1 Zone 2 sand (31%)

C2 Zone 2 sand (45%)

C3 Zone 2 sand (58%)

Table 5: Trial mixes for combined sands and basalt

Trial mix ratios	Range of values	Chosen mix
Water/cement ratio	0.48 – 0.51	0.5
Aggregate/cement ratio	3.2 – 3.4	3.4
Fine sand/total agg. ratio (%)	20 – 31	21
(Fine and coarse sand)/total agg. ratio (%)	67 – 80	70
Crushed basalt/total agg. ratio (%)	20 - 33	30

Table 6: Concrete strengths of control specimens

Model	Average cube compressive strength f_{cu} (Mpa)	Average split cylinder strength f_t (MPa)	$\frac{f_t}{\sqrt{f_{cu}}}$	$\frac{f_t}{(f_{cu})^{2/3}}$	$\frac{f_t}{f_{cu}}$
1	47.8	3.19	0.46	0.24	0.067
2	47.2	3.76	0.55	0.29	0.080
3	46.3	3.94	0.58	0.31	0.085
4	50.5	3.59	0.51	0.26	0.071
5	51.5	3.98	0.55	0.29	0.077
6	46.5	3.48	0.51	0.27	0.075
7	47.5	3.96	0.57	0.30	0.083
Coeff. of variation (%)			7.87	8.75	8.38
Prototype	41.6	3.55	0.55	0.30	0.086

Table 7: Loading rates for control specimens

Type of specimen	Load rate (kN/min)
150mm cube	340 (BS 1881)
100mm cube	150
50mm cube	38
150mm diam. cylinder	106 (BS 1881)
100mm diam. cylinder	47
50mm diam. cylinder	12

Discussion: It is obvious from Table 6 that for the limited amount of data covered in the present study, relationships of the type:

$$f_t = k_1(f_{cu})^{1/2} \quad (3)$$

$$f_t = k_2(f_{cu})^{2/3} \quad (4)$$

$$f_t = k_3 f_{cu} \quad (5)$$

where k_1 , k_2 and k_3 are constants all appear to be relatively satisfactory in describing the variation between the split tensile strength of the concrete, f_t , and the compressive cube strength, f_{cu} . In fact, the mean values of k_1 and k_2 from the test results (0.53 and 0.28 respectively) clearly demonstrate that the results are in reasonable agreement with those of earlier investigators [19-20].

In Table 6, the compressive and tensile strengths of the one-third scale models are generally higher than those of the prototype structure. This is consistent with

previous findings [29-30]. However, the investigations in [30] clearly demonstrate that these effects are more pronounced for model specimen sizes below the one-third scale used in the present study.

The importance of the rate of loading on the concrete strength has been recognized by several investigators [31-33] who found that an increase in the rate of loading normally resulted in higher strengths. Also it was noted that beyond a critical strain rate, large strength increases do occur [34]. Hence, scaled down loading rates were used for the 1/3 and 2/3 scale cubes and cylinders in the present investigations (see Table 7). Nevertheless, the tests on the 50mm x 100mm cylinders (for tensile strengths) required delicate handling in order to get consistent results. It is interesting to note that for such split cylinder tensile tests the use of a jig for alignment and positioning of the specimen in the testing machine is recommended [35].

Conclusion: The following conclusions can be drawn based on the work reported in the present study:

1. In investigations on punching shear in flat slabs, great care is needed in modeling the prototype concrete mix so as to obtain reliable estimates of the compressive and tensile strengths. Extensive trial tests using local aggregates may be necessary.
2. The use of two grades of sand – a BS 882 Zone 2 fine sand and a coarser variety, in conjunction with crushed basalt, yields a good means of obtaining representative model mixes. However, information regarding water-cement and aggregate-cement ratios for such model mixes must be obtained from standard prototype design and scaled down grading curves for the desired workability.
3. The compressive and tensile strengths of the model concrete are generally higher than those of the prototype. Relationships of similar form to those given in the ACI and CEB-FIP codes including a linear one are all relatively quite satisfactory in expressing the variability between the concrete split tensile and compressive cube strengths.
4. Scaled down loading rates are necessary for the testing of model cubes and cylinders in order to obtain representative results.

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